

۲۵ عالی نوشتی

$$f \circ f \left(\frac{1}{\sqrt{3}} \right) = 1 \quad f(x) = \begin{cases} \cot + \frac{\pi x}{\varepsilon} & x \leq 1 \\ \sqrt{x^2 + 1} & x > 1 \end{cases}$$

$$f(f(x)) = f \circ f \left(\frac{1}{\sqrt{3}} \right) \quad f \left(\frac{1}{\sqrt{3}} \right) = \cot + \frac{\pi}{\varepsilon} = \cot + \frac{\pi}{\varepsilon} = \cot + 30^\circ = \sqrt{3}$$

$$\sqrt{3} = x \Rightarrow x > 1$$

$$\frac{1}{\sqrt{3}} = x \Rightarrow x \leq 1$$

$$f(\sqrt{3}) = \sqrt{(\sqrt{3})^2 + 1} = \sqrt{4} = 2$$

$$f \circ f \left(\frac{1}{\sqrt{3}} \right) = 2 \quad \checkmark$$

$$\omega) f \circ g \left(\frac{\pi}{\varepsilon} \right) \quad g(x) = 2 \cos^2 x \quad f \left(\frac{1}{x} \right) = \sqrt{\frac{2x-1}{x^2}} \Rightarrow f(x) = \sqrt{\frac{x-1}{x}} = \sqrt{\frac{x-1}{x}} = \sqrt{2-x} \quad \checkmark$$

$$\Rightarrow f \circ g \left(\frac{\pi}{\varepsilon} \right) = f \left(g \left(\frac{\pi}{\varepsilon} \right) \right) \Rightarrow f \left(\frac{1}{\varepsilon} \right) = \sqrt{2 \times \frac{1}{\varepsilon} - \frac{1}{\varepsilon}} = \sqrt{\frac{1}{\varepsilon}} \quad \checkmark$$

$$g \left(\frac{\pi}{\varepsilon} \right) = 2 \cos^2 \frac{\pi}{\varepsilon} = 2 \cos^2 60^\circ = \frac{1}{\varepsilon} \times 2 = \frac{1}{\varepsilon}$$

$$\text{ب) } f \circ g(x) = \left[\frac{x}{1-x} \right] \rightarrow f \circ g(\sqrt{2}) = \left[\frac{\sqrt{2}}{1-\sqrt{2}} \right] = \left[\frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} \right] = \left[\frac{\sqrt{2}+2}{-1} \right] = [-2-\sqrt{2}] = -\varepsilon \quad \checkmark$$

$$g \circ f \left(\frac{\pi}{\varepsilon} \right) = ? \quad g(x) = x \sqrt{1-x^2} \quad f(x) = \sin x$$

$$g \circ f(x) = \sin x \sqrt{1-\sin^2 x} \Rightarrow g \circ f \left(\frac{\pi}{\varepsilon} \right) = \sin \frac{\pi}{\varepsilon} \sqrt{1-\sin^2 \frac{\pi}{\varepsilon}} = \sin \varepsilon \sqrt{1-\sin^2 \varepsilon}$$

$$\Rightarrow g \circ f \left(\frac{\pi}{\varepsilon} \right) = \frac{\sqrt{2}}{\varepsilon} \times \sqrt{1-\frac{1}{\varepsilon}} = \frac{\sqrt{2}}{\varepsilon} \times \frac{1}{\sqrt{2}} = \frac{1}{\varepsilon} \quad \checkmark$$

$$g(x) = \{(x, y), (y, x), (x, z), (z, x)\} \quad f(x) = \{(x, y), (y, x), (x, z), (z, x)\}$$

$$\omega) f \circ g(x) \Rightarrow \begin{matrix} x & \xrightarrow{g(x)} & y & \xrightarrow{f(x)} & z \\ y & \xrightarrow{g(x)} & x & \xrightarrow{f(x)} & z \\ z & \xrightarrow{g(x)} & x & \xrightarrow{f(x)} & y \\ x & \xrightarrow{g(x)} & y & \xrightarrow{f(x)} & x \end{matrix}$$

$$\Rightarrow f \circ g(x) = \{(x, z), (y, z), (x, y), (y, x)\} \quad \checkmark$$

$$\text{ج) } f \circ f(x) \Rightarrow \begin{matrix} x & \xrightarrow{f(x)} & y & \xrightarrow{f(x)} & x \\ y & \xrightarrow{f(x)} & x & \xrightarrow{f(x)} & y \\ z & \xrightarrow{f(x)} & x & \xrightarrow{f(x)} & y \\ x & \xrightarrow{f(x)} & y & \xrightarrow{f(x)} & x \end{matrix} \Rightarrow f \circ f(x) = \emptyset \quad \checkmark$$

$$\text{ب) } g \circ f(x) \Rightarrow \begin{matrix} x & \xrightarrow{f(x)} & y & \xrightarrow{g(x)} & x \\ y & \xrightarrow{f(x)} & x & \xrightarrow{g(x)} & y \\ z & \xrightarrow{f(x)} & x & \xrightarrow{g(x)} & y \\ x & \xrightarrow{f(x)} & y & \xrightarrow{g(x)} & x \end{matrix}$$

$$\Rightarrow g \circ f(x) = \emptyset \quad \checkmark$$

$$\text{د) } g \circ g(x) \Rightarrow \begin{matrix} x & \xrightarrow{g(x)} & y & \xrightarrow{g(x)} & x \\ y & \xrightarrow{g(x)} & x & \xrightarrow{g(x)} & y \\ z & \xrightarrow{g(x)} & x & \xrightarrow{g(x)} & y \\ x & \xrightarrow{g(x)} & y & \xrightarrow{g(x)} & x \end{matrix}$$

$$g \circ g(x) = \{(x, y), (y, x), (x, z), (z, x)\} \quad \checkmark$$

$$g = \{(1, 2), (2, 1), (a, 3), (b, 1)\} \quad f = \{(2, 1), (3, 2), (4, 5), (1, 7)\} \quad (4, 2) \in f \circ g \quad (4, 1) \in g \circ f$$

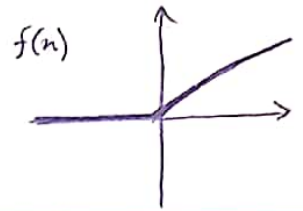
$$f \circ g(x) = \begin{matrix} 1 & \rightarrow & 2 & \rightarrow & 1 \\ 2 & \rightarrow & 1 & \rightarrow & 7 \\ a & \rightarrow & 3 & \rightarrow & 2 \\ b & \rightarrow & 1 & \rightarrow & 7 \end{matrix} \Rightarrow f \circ g(x) = \{(1, 1), (2, 7), (a, 2), (b, 7)\}$$

$$(a, b) = (4, 5) \quad \checkmark$$

$$g \circ f(x) = \begin{matrix} 2 & \rightarrow & 1 & \rightarrow & 2 \\ 4 & \rightarrow & 5 & \rightarrow & 1 \\ 1 & \rightarrow & 7 & \rightarrow & 2 \end{matrix} \Rightarrow g \circ f(x) = \{(2, 2), (4, 1), (1, 2)\} \Rightarrow b = 5 \quad \checkmark$$

$f(f(x)) = \{x+3\} \Rightarrow g(2x+3) = 3x-2 \rightarrow 2x+3 = t \Rightarrow x = \frac{t-3}{2} \Rightarrow$ جایگزینی
 $f(x) = ax+b \Rightarrow \begin{cases} g(2x+3) = 3x-2 \\ 3\left(\frac{t-3}{2}\right) - 2 = \frac{3t-9}{2} - 2 = \frac{3t-13}{2} \Rightarrow g(x) = \frac{3x-13}{2} \end{cases}$
 $f(f(x)) = a(ax+b) + b = a^2x + ab + b = \{x+3\} \Rightarrow a^2 = 1 \Rightarrow a = \pm 1$
 if $a=2 \Rightarrow 2b+b=3 \Rightarrow b=1 \Rightarrow f(x) = 2x+1 \Rightarrow f(-1) = -1$
 if $a=-2 \Rightarrow -2b+b=3 \Rightarrow b=-3 \Rightarrow f(x) = -2x-3 \Rightarrow f(-1) = -1$
 $g \circ f(-1) = g(f(-1)) \Rightarrow g(-1) \Rightarrow g(-1) = \frac{-3-13}{2} = \frac{-16}{2} = -8 \checkmark$

$f(x) = \sqrt{x+|x|} \quad g(x) = \frac{1}{x^2 - \varepsilon x} \quad D_{g \circ f} = \{x \in D_f : f(x) \in D_g\}$
 $D_f = x+|x| > 0 \Rightarrow \begin{cases} \text{if } x \leq 0 \Rightarrow x-x \geq 0 \Rightarrow 0 \geq 0 \checkmark \\ \text{if } x > 0 \Rightarrow x+x \geq 0 \Rightarrow 2x \geq 0 \Rightarrow x \geq 0 \checkmark \end{cases} \Rightarrow D_f = \mathbb{R}$
 $D_g = x^2 - \varepsilon x \neq 0 \Rightarrow x^2 - \varepsilon x = 0 \Rightarrow x(x - \varepsilon) = 0 \Rightarrow x=0, \varepsilon \Rightarrow D_g = \mathbb{R} - \{0, \varepsilon\}$
 $\sqrt{x+|x|} = 0 \Rightarrow x+|x| = 0 \Rightarrow x \leq 0$ در ازای این مقدار عبارت صفر برابر صفر است
 $\sqrt{x+|x|} = \varepsilon \Rightarrow x+|x| = \varepsilon^2 \Rightarrow x > 0 \Rightarrow 2x = \varepsilon^2 \Rightarrow x = \frac{\varepsilon^2}{2}$ نباید باشد
 $\Rightarrow D_{g \circ f} = (0, +\infty) - \{\frac{\varepsilon^2}{2}\} \checkmark$



$f(x) = \sqrt{1-x^2} \quad g(x) = \sqrt{x} \quad D_{(f+g) \circ f} = \{x \in D_f : f(x) \in D_{(f+g)}\}$
 $(f+g) = f(x) + g(x) = \sqrt{x} + \sqrt{1-x^2} \Rightarrow \begin{cases} 1-x^2 \geq 0 \Rightarrow |x| \leq 1 \\ x \geq 0 \end{cases} \Rightarrow D_{(f+g)} = [0, 1]$
 $D_f = 1-x^2 \geq 0 \Rightarrow |x| \leq 1 \Rightarrow x \in [-1, 1]$
 $f(x) \in D_{(f+g)} \Rightarrow 0 \leq \sqrt{1-x^2} \leq 1 \Rightarrow 0 \leq 1-x^2 \leq 1 \Rightarrow -1 \leq -x^2 \leq 0 \Rightarrow 0 \leq x^2 \leq 1 \Rightarrow -1 \leq x \leq 1$
 $\Rightarrow I \cap II \Rightarrow D_{(f+g) \circ f} = [-1, 1] \checkmark$

الف) $f\left(\frac{3x+1}{x-2}\right) = \varepsilon x + \omega \Rightarrow \frac{3x+1}{x-2} = t \Rightarrow 3x+1 = tx-2t \Rightarrow 1+2t = tx-3x \Rightarrow 1+2t = x(t-3) \Rightarrow x = \frac{1+2t}{t-3}$
 $\varepsilon x + \omega = \varepsilon \left(\frac{1+2t}{t-3}\right) + \omega = \frac{\varepsilon + 2\varepsilon t}{t-3} + \omega$
 $\Rightarrow \varepsilon x + \omega = \frac{\varepsilon + 2\varepsilon t + \omega(t-3)}{t-3} = \frac{13t-11}{t-3} \Rightarrow f(x) = \frac{13x-11}{x-3} \checkmark$
 ب) $f\left(x + \frac{1}{x}\right) = x^3 + \frac{1}{x^3} \Rightarrow x + \frac{1}{x} = t \Rightarrow f(t) = x^3 + \frac{1}{x^3} + 3x + \frac{3}{x} - \left(3x + \frac{3}{x}\right)$
 $\Rightarrow f(t) = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right) \Rightarrow f(t) = t^3 - 3t \Rightarrow f(x) = x^3 - 3x \checkmark$

$g(x) \rightarrow$ برای x برابر با $2\sqrt{x}$ و عدد x کسری قطع می کنند
 $g \circ f = g(f(x)) \Rightarrow f(x) = x\sqrt{x}$
 $f(x) = 2\sqrt{x} \Rightarrow x\sqrt{x} = 2\sqrt{x} \Rightarrow \sqrt{x^3} = \sqrt{4} \Rightarrow x^3 = 4 \Rightarrow x = \sqrt[3]{4} \checkmark$
 $f(x) = 1 \Rightarrow x\sqrt{x} = 1 \Rightarrow \sqrt{x^3} = \sqrt{1} \Rightarrow 1 = x^3 \Rightarrow x = 1 \checkmark$
 اختلاف خواسته طول شماره سوال $= 2-1=1 \checkmark$