محمد امین جعفری　　　تکلیف شماره ۹　　　یازدهم تجربی B

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$\cot\frac{\pi}{4}\times\frac{2}{3}=\cot\frac{\pi}{6}=\sqrt{3}=f(\frac{2}{3})$

$f(\sqrt{3})=\sqrt{x^2+1}=\sqrt{3+1}=\sqrt{4}=2$

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الف)

$g(\frac{\pi}{3})=2\cos^2\frac{\pi}{3}=2(\frac{1}{2})^2=\frac{1}{2}$

$f(\frac{1}{2})=\sqrt{\frac{2(2)-1}{2^2}}=\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$

ب)

$g(\sqrt{2})=\frac{\sqrt{2}}{1-\sqrt{2}}=\frac{\sqrt{2}(1+\sqrt{2})}{1-2}=-\sqrt{2}(1+\sqrt{2})=-2-\sqrt{2}$

$f(-2-\sqrt{2})=[-2-\sqrt{2}]=-4$

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$f(\frac{\pi}{4})=\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}=\frac{1}{\sqrt{2}}$

$g(\frac{1}{\sqrt{2}})=\frac{1}{\sqrt{2}}\sqrt{1-(\frac{1}{\sqrt{2}})^2}=\frac{1}{\sqrt{2}}\sqrt{\frac{1}{2}}=\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}=\frac{1}{2}$

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الف) $\{(4,5)(6,13)(8,12)(2,5)\}$

ب) $g\circ f=\varnothing$

ج) $f\circ f=\varnothing$

د) $\{(4,8)(2,6)(6,10)\}$

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$f(g(x))=2\Rightarrow g(x)=4\Rightarrow x=a=4$

$t=4\Rightarrow f(t)=5\Rightarrow g(f(t))=g(5)=1\Rightarrow b=5$

$\Rightarrow (a,b)=(4,5)$

$g(2x+3) = 3x-2 = a(2x+3)+b = 2ax+3a+b \Rightarrow \begin{matrix} a = \frac{3}{2} \\ b = -\frac{13}{2} \end{matrix} \Rightarrow g(x) = \frac{3}{2}x - \frac{13}{2}$ 7

$f(f(x)) = 4x+3 = a'(f(x))+b' = a'(a'x+b')+b' = a'^2 x + a'b' + b' \Rightarrow \begin{cases} a'=2 \Rightarrow b'=1 \\ a'=-2 \Rightarrow b'=-3 \end{cases}$

$\Rightarrow f(x) = \begin{cases} 2x+1 \Rightarrow f(-1) = -2+1 = -1 \\ -2x-3 \Rightarrow f(-1) = 2-3 = -1 \end{cases} \Rightarrow g(-1) = -\frac{3}{2} - \frac{13}{2} = -8$

$f(x) \in D_g \quad x^2-4x = x(x-4) \neq 0 \Rightarrow x \neq \begin{cases} 0 \\ 4 \end{cases} \quad D_g = \mathbb{R} - \{0, 4\}$

$\Rightarrow f(x) \neq \left.\begin{cases} 0 \Rightarrow x \neq 0 \\ 4 \Rightarrow x + |x| \neq 16 \Rightarrow x \neq 8 \end{cases}\right\} \Rightarrow D_f = \mathbb{R} - \{0, 8\}$

$x \in D_f \Rightarrow x \in \mathbb{R} - \{0, 8\}$

$f(x) \in D_{f+g} \quad (f+g)(x) = \sqrt{x} + \sqrt{1-x^2} \begin{cases} 1-x^2 \geq 0 \Rightarrow x^2 \leq 1 \Rightarrow -1 \leq x \leq 1 \ (1) \\ x > 0 \ (2) \end{cases}$ 8

$(1)\cap(2) \Rightarrow 0 < x \leq 1 = D_{f+g} \Rightarrow 0 < f(x) \leq 1 \Rightarrow 0 < \sqrt{1-x^2} \leq 1 \Rightarrow 0 < 1-x^2 \leq 1$

$1-x^2 \geq 0 \Rightarrow x^2 \leq 1 \Rightarrow -1 \leq x \leq 1 \ (3)$

$-1 < -x^2 \leq 0 \Rightarrow 0 \leq x^2 < 1 \Rightarrow -1 < x < 1 \ (4)$

$(3)\cap(4) \Rightarrow -1 < D_f < 1 \qquad x \in D_f \Rightarrow -1 < x < 1$

الف) 9

$\frac{2x+1}{x-2} = t \Rightarrow x = \frac{-2t-1}{-t+3} \Rightarrow 4x+5 = \frac{-8t-4-5t+15}{-t+3} = \frac{-13t+11}{-t+3}$

$\Rightarrow f(t) = \frac{-13t+11}{-t+3}$

ب)

$x^2 + \frac{1}{x^2} = \left(x+\frac{1}{x}\right)^2 - 2\left(x+\frac{1}{x}\right) \Rightarrow f(t) = t^2 - 2t$

$g(x) = 0 \Rightarrow x = \begin{cases} 1 \\ 2\sqrt{2} \end{cases}$ 10

$f(x) = x\sqrt{x} \begin{cases} = 2\sqrt{2} \Rightarrow x = 2 \\ = 1\sqrt{1} \Rightarrow x = 1 \end{cases} \qquad 2-1 = 1$