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$$f(x) = ax + b \Rightarrow f(f(x)) = a(ax + b) + b = a^2x + ab + b$$

$$a^2x + ab + b = 4x + 3 \Rightarrow a = 2 \quad b = 3 \Rightarrow b = 1$$

$$2x + 3 = 4 \quad x = \frac{4-3}{2} \quad y\left(\frac{4}{2}\right) = 3\left(\frac{4-3}{2}\right) - 2 = \frac{3(4-9)}{2} - 2 \Rightarrow g(x) = \frac{3x-9}{2}$$

$$g \circ f(-1) = g(2(-1) + 1) = g(-1) = \frac{3(-1) - 9}{2} - 2 = \frac{-12}{2} - 2 = -6 - 2 = -8$$

سوال ۶

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$$g \circ f\left(\frac{\pi}{4}\right) = g\left(\sin \frac{\pi}{4}\right) = g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \sqrt{1 - \frac{1}{2}} = \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$$

سوال ۳

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$$a) f \circ g\left(\frac{\pi}{4}\right) = f(2 \cos^2 \frac{\pi}{4}) = f\left(\frac{1}{2}\right) = \sqrt{\frac{1}{2} - 1} = \frac{\sqrt{3}}{2}$$

$$b) f \circ g(\sqrt{2}) = f\left(\frac{\sqrt{2}}{1 - \sqrt{2}}\right) = \left[\frac{\sqrt{2}}{1 - \sqrt{2}}\right] = -4$$

سوال ۲

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$$a) f \circ g(x) = \{(4, 4), (2, 5), (6, 12), (8, 12)\}$$

$$b) g \circ f(x) = \emptyset$$

$$c) f \circ f(x) = \emptyset$$

$$d) g \circ g(x) = \{(4, 8), (2, 4), (6, 10)\}$$

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$$f \circ g(4) = 2 \quad f(3) = 2 \Rightarrow g(4) = 3 \Rightarrow a = 3$$

$$g \circ f(4) = 1 \quad g(f(4)) = 1 \Rightarrow g(4) = 1 \Rightarrow b = 4$$

$$(a, b) = (3, 4)$$

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$$f \circ f\left(\frac{1}{f}\right) = f\left(f\left(\frac{1}{f}\right)\right) = f\left(\cot \frac{\pi}{4} \times \frac{1}{f}\right) = f\left(\cot \frac{\pi}{4}\right) = f(\sqrt{f})$$

$$= \sqrt{(\sqrt{f})^2 + 1} = \sqrt{f+1} = 2 \quad \checkmark$$

سوال ۱

$$D_f = \mathbb{R} \quad D_g = \mathbb{R} - \{0, 4\} \Rightarrow f \neq 0, 4$$

$$\frac{1}{2(x-4)}$$

$$f(x) \begin{cases} x > 0 \Rightarrow f = \sqrt{2x} \Rightarrow x \neq 1 \\ x \leq 0 \Rightarrow f = 0 \quad \times \end{cases} \Rightarrow D_{g \circ f} = (0, +\infty) - \{1\} \quad \checkmark$$

$$D_f: 1 - x^2 \geq 0 \Rightarrow x^2 \leq 1 \Rightarrow -1 \leq x \leq 1 \Rightarrow R_f = [0, 1]$$

$$D_{g \circ f} \subset D_f \longrightarrow \text{زیرا برد تابع f همواره نامنفی است.}$$

$$D_f = [-1, 1] \quad \checkmark$$

$$D_{f \circ f} = D_f \longrightarrow \text{زیرا برد تابع f همواره در بازه دامنه تابع f قرار میگیرد.}$$

$$\text{ا) } t = \frac{3x+1}{x-2} \Rightarrow x = \frac{t+1}{t-3} \Rightarrow f(t) = \frac{1+t+t^2}{t-3} + 4 = \frac{t^2-1}{t-3}$$

$$\Rightarrow f(x) = \frac{x^2-1}{x-3} \quad \checkmark$$

$$\text{ب) } x + \frac{1}{x} = t \Rightarrow x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2(1) = t^2 - 2 \Rightarrow f(t) = t^2 - 2$$

$$\Rightarrow f(x) = x^2 - 2 \quad \checkmark$$

$$x\sqrt{x} = 1 \Rightarrow x = 1$$

$$x\sqrt{x} = 2\sqrt{2} \Rightarrow x = 2$$

$$2-1 = 1 \quad \checkmark$$