

$$f(f(\frac{1}{e})) \Rightarrow \frac{1}{e} < 1 \Rightarrow \cot \frac{2\pi}{11} = \cot \frac{\pi}{5} = \sqrt{5} = f(\frac{1}{e})$$

$$\Rightarrow f(\sqrt{5}) \Rightarrow \sqrt{5} > 1 \Rightarrow \sqrt{(\sqrt{5})^2 + 1} = \sqrt{6} = 2$$

$$f \circ f(\frac{1}{e}) = 2$$

$$f(g(\frac{\pi}{e})) \Rightarrow g(\frac{\pi}{e}) = 2 \cos \frac{\pi}{e} = \frac{1}{e} \Rightarrow f(\frac{1}{e}) = \sqrt{\frac{1^2 - 1}{e^2}} = \frac{\sqrt{2}}{e}$$

$$f(g(\sqrt{2})) \Rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \xrightarrow{\text{تواند}} \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{-1} = -2-\sqrt{2}$$

$$f(-2-\sqrt{2}) = [-2-\sqrt{2}] = -4$$

$$g(f(\frac{\pi}{e})) \Rightarrow f(\frac{\pi}{e}) = \sin \frac{\pi}{e} = \frac{\sqrt{2}}{e}$$

$$g(\frac{\sqrt{2}}{e}) = \frac{\sqrt{2}}{e} \sqrt{1 - (\frac{\sqrt{2}}{e})^2} \Rightarrow \frac{\sqrt{2}}{e} \times \frac{\sqrt{2}}{e} = \frac{1}{e}$$

$$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} \text{ تواند}$$

$$f \circ g(n) = \{(f, \omega), (2, \omega), (5, 12), (1, 12)\}$$

$g(g(n)) \Rightarrow g(4) = 5$ (د)	$f(f(n))$ (ز)	$g(f(n))$ (ب)	$f(g(n))$ (ان)
$g(4) = 5 \rightarrow f \circ g(4) = 5$	$f(4) = \omega \rightarrow$ در دایره	$f(4) = \omega \rightarrow$ در دایره	$\Rightarrow g(4) = 5 \rightarrow f(5) = \omega$
$g(2) = 4 \rightarrow g(4) = 5 \rightarrow f \circ g(2) = 5$	$f(5) = \omega \rightarrow \sim$	$f(2) = \omega \rightarrow$ در دایره	$f \circ g(4) = \omega$
$g(5) = 1 \rightarrow g(1) = 10 \rightarrow f \circ g(5) = 10$	$f(1) = 12 \rightarrow \sim$	$f(1) = 12 \rightarrow \sim$	$g(2) = 4 \rightarrow f(4) = \omega$
$g(1) = 10 \rightarrow$ در دایره $f \circ g(1) = 10$	$f(10) = 12 \rightarrow \sim$	$f(10) = 12 \rightarrow \sim$	$f \circ g(2) = \omega$
$g \circ g(n) = \{(4, 1), (2, 5), (5, 10)\}$	$f \circ f(n) = \emptyset$	$g \circ f(n) = \emptyset$	$g(5) = 1 \rightarrow f(1) = 12 \rightarrow f \circ g(5) = 12$
			$g(1) = 10 \rightarrow f(10) = 12 \rightarrow f \circ g(1) = 12$

$$f(g(n)) \Rightarrow f(g(4)) = 2 \Rightarrow f(\omega) = 2 \Rightarrow \omega = 2$$

$$g(4) = 2 \Rightarrow (a, 2) = (4, 2) \Rightarrow a = 4$$

$$g(f(n)) \Rightarrow g(f(4)) = 1 \Rightarrow f(4) = \omega \Rightarrow g(\omega) = 1 \Rightarrow (b, 1) = (\omega, 1)$$

$$= b = \omega \Rightarrow (a, b) = (4, \omega)$$

$$f(f(n)) = 5n + 2 \Rightarrow f(n) = an + b \Rightarrow f(an + b) = a(an + b) + b = 5n + 2$$

$$\Rightarrow a^2 = 5 \Rightarrow a = \pm\sqrt{5} \rightarrow a = -\sqrt{5} \Rightarrow b = -\frac{2}{\sqrt{5}} \quad \left\{ \begin{array}{l} f(n) = y = 5n + 2 \\ f(n) = y = -5n - 2 \end{array} \right. \quad \frac{5t}{5} - \frac{15}{5} = g(t)$$

$$g(5n+2) = 5n-2 \Rightarrow g(t) = 5n-2 \Rightarrow t = 5n+2 \Rightarrow n = \frac{t-2}{5} \Rightarrow g(t) = 5\left(\frac{t-2}{5}\right) - 2$$

$$\Rightarrow g(n) = \frac{5}{5}n - 2 = n - 2 \Rightarrow g(f(-1)) = f(-1) = -1 \Rightarrow g(-1) = -1 - 2 = -3 = g \circ f(-1)$$

$$g(f(n)) \Rightarrow n \in D_f \Rightarrow D_f = \mathbb{R}, f(n) \in D_g \Rightarrow D_g = \mathbb{R} \setminus \{0, 5\}$$

$$5n - 5n \neq 0 \Rightarrow n \neq 0, 5 \Rightarrow D_g = \mathbb{R} - \{0, 5\} \Rightarrow f(n) \neq 0, 5$$

$$f(n) \neq 0 \Rightarrow (-\infty, 0] f(n) \neq 5 \Rightarrow \sqrt{5n+2} \neq 5 \Rightarrow n+1 \neq 15$$

$$5n = 15 \Rightarrow n = 3 \Rightarrow D_{g \circ f} = \mathbb{R} - \{3\} - (-\infty, 0] = (0, +\infty) - \{3\}$$

$$f+g(f(n)) \Rightarrow n \in D_f \Rightarrow D_f \Rightarrow 1-n^2 > 0 \Rightarrow -1 < n < 1$$

$$D_f = [-1, 1] \quad f(n) \in D_{f+g} \Rightarrow D_{f+g} = D_f \cap D_g \quad D_g = n > 0$$

$$D_{f+g} = [0, 1] \Rightarrow 0 \leq \sqrt{1-n^2} \leq 1 \Rightarrow 0 \leq n^2 \leq 1 \Rightarrow -1 \leq n \leq 1$$

$$\Rightarrow D_{f+g(f(n))} = [-1, 1]$$

$$f\left(\frac{5n+1}{n-5}\right) = 5n+2 \Rightarrow f(t) = 5n+2 \Rightarrow t = \frac{5n+1}{n-5} \Rightarrow n = -\frac{5t-1}{t-5}$$

$$\Rightarrow n = \frac{5t+1}{t-5} \Rightarrow f(t) = 5\left(\frac{5t+1}{t-5}\right) + 2 = \frac{25t+11}{t-5} \Rightarrow f(n) = \frac{15n-11}{n-5}$$

$$f\left(n + \frac{1}{n}\right) = n^2 + \frac{1}{n^5} \Rightarrow t = n + \frac{1}{n} \Rightarrow f(t) = \left(n + \frac{1}{n}\right)\left(n^2 + \frac{1}{n^5} - 1\right)$$

$$\Rightarrow f(t) = t\left(\frac{n^2+1}{n^2} - 1 + 5 - 5\right) \Rightarrow f(t) = t^2 - 5t \Rightarrow f(n) = n^2 - 5n$$

$$g(1) = 0 \quad g(\sqrt{2}) = 0 \Rightarrow g(f(n)) = 0 \Rightarrow f(n) = 1 \quad \text{or} \quad f(n) = \sqrt{2}$$

$$f(n) = n\sqrt{n} \Rightarrow n\sqrt{n} = 1 \Rightarrow n_1 = 1$$

$$n\sqrt{n} = 2\sqrt{2} \Rightarrow n_2 = 2 \quad \Rightarrow n_2 - n_1 = 2 - 1 = 1$$