

$f(f(\frac{1}{2})) \Rightarrow \frac{1}{2} < 1 \Rightarrow \cot \frac{2\pi}{12} = \cot \frac{\pi}{6} = \sqrt{3} = f(\frac{1}{2})$ $\Rightarrow f(\sqrt{3}) \Rightarrow \sqrt{3} > 1 \Rightarrow \sqrt{(\sqrt{3})^2 + 1} = \sqrt{4} = 2$ $f \circ f(\frac{1}{2}) = 2 \quad \checkmark$	۲																												
$f(g(\frac{\pi}{4})) \Rightarrow g(\frac{\pi}{4}) = 2 \cos \frac{\pi}{4} = \frac{1}{2} \Rightarrow f(\frac{1}{2}) = \sqrt{\frac{2 \times 2 - 1}{2^2}} = \frac{\sqrt{2}}{2} \quad \checkmark$ $f(g(\sqrt{2})) \Rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \xrightarrow{\text{تواند}} \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{-1} = -2-\sqrt{2}$ $f(-2-\sqrt{2}) = [-2-\sqrt{2}] = -4 \quad \checkmark$	۲																												
$g(f(\frac{\pi}{4})) \Rightarrow f(\frac{\pi}{4}) = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2} \sqrt{1 - (\frac{\sqrt{2}}{2})^2} \Rightarrow \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{1}{2} \quad \checkmark$ $\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} \quad \text{تواند}$ $f \circ g(n) = \{(4, \omega), (2, \omega), (5, 12), (1, 12)\} \quad \checkmark$	۳																												
<table><tr><th>$g(g(n)) \Rightarrow g(4) = 8$</th><th>$f(f(n))$</th><th>$g(f(n))$</th><th>$f(g(n))$</th></tr><tr><td>$g(4) = 8 \rightarrow f \circ g(4) = 8$</td><td>$f(4) = \omega \rightarrow$ در دایره</td><td>$f(4) = \omega \rightarrow$ در دایره</td><td>$\Rightarrow g(4) = 8 \rightarrow f(8) = \omega$</td></tr><tr><td>$g(2) = 4 \rightarrow g(4) = 8 \rightarrow f \circ g(2) = 8$</td><td>$f(8) = \omega \rightarrow \sim$</td><td>$f(2) = \omega \rightarrow$ در دایره</td><td>$f \circ g(4) = \omega$</td></tr><tr><td>$g(5) = 1 \rightarrow g(1) = 10 \rightarrow f \circ g(5) = 10$</td><td>$f(1) = 12 \rightarrow \sim$</td><td>$f(1) = 12 \rightarrow \sim$</td><td>$g(2) = 4 \rightarrow f(4) = \omega$</td></tr><tr><td>$g(1) = 10 \rightarrow$ در دایره $f \circ g(1) = 10$</td><td>$f(10) = 12 \rightarrow \sim$</td><td>$f(10) = 12 \rightarrow \sim$</td><td>$f \circ g(2) = \omega$</td></tr><tr><td>$g \circ g(n) = \{(4, 8), (2, 4), (5, 10)\}$</td><td>$f \circ f(n) = \emptyset \quad \checkmark$</td><td>$g \circ f(n) = \emptyset \quad \checkmark$</td><td>$g(5) = 1 \rightarrow f(1) = 12 \rightarrow f \circ g(5) = 12$</td></tr><tr><td></td><td></td><td></td><td>$g(1) = 10 \rightarrow f(10) = 12 \rightarrow f \circ g(1) = 12$</td></tr></table>	$g(g(n)) \Rightarrow g(4) = 8$	$f(f(n))$	$g(f(n))$	$f(g(n))$	$g(4) = 8 \rightarrow f \circ g(4) = 8$	$f(4) = \omega \rightarrow$ در دایره	$f(4) = \omega \rightarrow$ در دایره	$\Rightarrow g(4) = 8 \rightarrow f(8) = \omega$	$g(2) = 4 \rightarrow g(4) = 8 \rightarrow f \circ g(2) = 8$	$f(8) = \omega \rightarrow \sim$	$f(2) = \omega \rightarrow$ در دایره	$f \circ g(4) = \omega$	$g(5) = 1 \rightarrow g(1) = 10 \rightarrow f \circ g(5) = 10$	$f(1) = 12 \rightarrow \sim$	$f(1) = 12 \rightarrow \sim$	$g(2) = 4 \rightarrow f(4) = \omega$	$g(1) = 10 \rightarrow$ در دایره $f \circ g(1) = 10$	$f(10) = 12 \rightarrow \sim$	$f(10) = 12 \rightarrow \sim$	$f \circ g(2) = \omega$	$g \circ g(n) = \{(4, 8), (2, 4), (5, 10)\}$	$f \circ f(n) = \emptyset \quad \checkmark$	$g \circ f(n) = \emptyset \quad \checkmark$	$g(5) = 1 \rightarrow f(1) = 12 \rightarrow f \circ g(5) = 12$				$g(1) = 10 \rightarrow f(10) = 12 \rightarrow f \circ g(1) = 12$	۴
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$f(g(n)) \Rightarrow f(g(4)) = 2 \Rightarrow f(\omega) = 2 \Rightarrow \omega = 2$ $g(4) = 2 \Rightarrow (a, c) = (4, 2) \Rightarrow a = 4 \quad \checkmark$ $g(f(n)) \Rightarrow g(f(4)) = 1 \Rightarrow f(4) = \omega \Rightarrow g(\omega) = 1 \Rightarrow (b, a) = (\omega, 1)$ $= b = \omega \quad \checkmark \Rightarrow (a, b) = (4, \omega) \quad \checkmark$	۵																												

$$f(f(n)) = 5n + 2 \Rightarrow f(n) = an + b \Rightarrow f(an + b) = a(an + b) + b = \frac{5n+2}{2}$$

$$\Rightarrow a^2 = \frac{5}{2} \Rightarrow a = \pm \sqrt{\frac{5}{2}} \rightarrow a = -\sqrt{\frac{5}{2}} \Rightarrow b = -\frac{2}{\sqrt{5}} \quad \{f(n) = y = 2n + 1\}$$

$$g(2n+1) = 2n-1 \Rightarrow g(t) = 2n-1 \Rightarrow t = 2n+1 \Rightarrow n = \frac{t-1}{2} \Rightarrow g(t) = 2\left(\frac{t-1}{2}\right) - 1 = t - 2$$

$$\Rightarrow g(n) = \frac{5n}{2} - 2 \Rightarrow g(f(-1)) = f(-1) = -1 \Rightarrow g(-1) = -\frac{5}{2} - 2 = -\frac{9}{2} = g \circ f(-1)$$

$$g(f(n)) \Rightarrow n \in D_f \Rightarrow D_f = \mathbb{R}, f(n) \in D_g \Rightarrow D_g = \mathbb{R} - \{0, 1\}$$

$$n^2 - 2n \neq 0 \Rightarrow n \neq 0, 2 \Rightarrow D_g = \mathbb{R} - \{0, 2\} \Rightarrow f(n) \neq 0, 2$$

$$f(n) \neq 0 \Rightarrow (-\infty, 0] f(n) \neq 1 \Rightarrow \sqrt{n+1} \neq 1 \Rightarrow n+1 \neq 1 \Rightarrow n \neq 0$$

$$2n = 1 \Rightarrow n = \frac{1}{2} \Rightarrow D_{g \circ f} = \mathbb{R} - \{1\} - (-\infty, 0] = (0, +\infty) - \{1\}$$

$$f+g(f(n)) \Rightarrow n \in D_f \Rightarrow D_f \Rightarrow 1 - n^2 > 0 \Rightarrow -1 < n < 1$$

$$D_f = [-1, 1] \quad f(n) \in D_{f+g} \Rightarrow D_{f+g} = D_f \cap D_g \quad D_g = n > 0$$

$$D_{f+g} = [0, 1] \Rightarrow 0 \leq \sqrt{1-n^2} \leq 1 \Rightarrow 0 \leq n^2 \leq 1 \Rightarrow -1 \leq n \leq 1$$

$$\Rightarrow D_{f+g(f(n))} = [-1, 1] \checkmark$$

$$f\left(\frac{2n+1}{n-1}\right) = 5n+2 \Rightarrow f(t) = 5n+2 \Rightarrow t = \frac{2n+1}{n-1} \Rightarrow n = -\frac{2t-1}{t-5}$$

$$\Rightarrow n = \frac{2t+1}{t-5} \Rightarrow f(t) = 5\left(\frac{2t+1}{t-5}\right) + 2 = \frac{12t-11}{t-5} \Rightarrow f(n) = \frac{12n-11}{n-5}$$

$$f\left(n + \frac{1}{n}\right) = n^2 + \frac{1}{n^2} \Rightarrow t = n + \frac{1}{n} \Rightarrow f(t) = \left(n + \frac{1}{n}\right)\left(n^2 + \frac{1}{n^2} - 1\right)$$

$$\Rightarrow f(t) = t\left(\frac{n^2+1}{n^2} - 1 + \frac{1}{n^2} - 1\right) \Rightarrow f(t) = t^2 - 2t \Rightarrow f(n) = n^2 - 2n \checkmark$$

$$g(1) = 0 \quad g(\sqrt{2}) = 0 \Rightarrow g(f(n)) = 0 \Rightarrow f(n) = 1 \quad f(n) = \sqrt{2}$$

$$f(n) = n\sqrt{n} \Rightarrow n\sqrt{n} = 1 \Rightarrow n_1 = 1 \checkmark$$

$$n\sqrt{n} = \sqrt{2} \Rightarrow n_2 = \sqrt{2} \checkmark \quad \Rightarrow n_2 - n_1 = \sqrt{2} - 1 = 1 \checkmark$$