

$$f(f(\frac{r}{r})) = ?$$

$$f(\frac{r}{r}) = 6 + \frac{\pi}{r-f} \times \frac{r}{r} = 6 + \frac{\pi}{4} = \sqrt{3}$$

$$\Rightarrow f(\frac{r}{r}) = \sqrt{3}$$

$$f \circ f(\frac{r}{r}) = f(f(\frac{r}{r})) = f(\sqrt{3}) = \sqrt{(\sqrt{3})^2 + 1} = (2)$$

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$$f(g(\sqrt{r})) = ?$$

(ب)

$$g(\sqrt{r}) = \frac{\sqrt{r}}{1-\sqrt{r}} \approx \frac{1.4}{1-1.4} = \frac{1.4}{-0.4} = -3.5$$

$$f(-3.5) = [-3.5] = (-3)$$

$$f \circ g(\frac{\pi}{r}) = f(g(\frac{\pi}{r})) = ?$$

(الف)

$$g(\frac{\pi}{r}) = 2 \times \cos^2 \frac{\pi}{r} - \frac{\cos \frac{\pi}{r}}{r} = 2 \times \frac{1}{4} = \frac{1}{2}$$

$$f(g(\frac{\pi}{r})) = f(\frac{1}{2}) \Rightarrow \frac{1}{x} = \frac{1}{r} \Rightarrow x = r$$

$$f(\frac{1}{r}) = \sqrt{\frac{r(r)-1}{r^2}} = \sqrt{\frac{r}{r}} = \boxed{\frac{\sqrt{r}}{r}}$$

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$$g \circ f(\frac{\pi}{r}) = g(f(\frac{\pi}{r})) = ?$$

$$f(\frac{\pi}{r}) = \sin \frac{\pi}{r} = \frac{\sqrt{r}}{r}$$

$$\Rightarrow g(\frac{\sqrt{r}}{r}) = \frac{\sqrt{r}}{r} \sqrt{1 - (\frac{\sqrt{r}}{r})^2} = \frac{\sqrt{r}}{r} \sqrt{1 - \frac{1}{r}} = \frac{\sqrt{r}}{r} \sqrt{\frac{r-1}{r}} = \frac{\sqrt{r-1}}{r} \times \frac{1}{\sqrt{r}} = \boxed{\frac{1}{r}}$$

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$$r \xrightarrow{g} 4 \xrightarrow{g} 8$$

$$2 \xrightarrow{g} 4 \xrightarrow{g} 4$$

$$4 \xrightarrow{g} 8 \xrightarrow{g} 10$$

$$8 \xrightarrow{g} 10 \xrightarrow{g} x$$

$$g \circ g(x) = \{(r, 8), (4, 4), (4, 10)\}$$

$$r \xrightarrow{f} 5 \xrightarrow{f} x$$

$$4 \xrightarrow{f} 5 \xrightarrow{f} x$$

$$8 \xrightarrow{f} 10 \xrightarrow{f} x$$

$$10 \xrightarrow{f} 10 \xrightarrow{f} x$$

$$f \circ f(x) = \emptyset$$

(ج)

$$r \xrightarrow{f} 5 \xrightarrow{g} x$$

$$4 \xrightarrow{f} 5 \xrightarrow{g} x$$

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$$g \circ f(x) = \emptyset$$

(ب)

$$r \xrightarrow{g} 4 \xrightarrow{f} 5$$

$$2 \xrightarrow{g} 4 \xrightarrow{f} 5$$

$$4 \xrightarrow{g} 8 \xrightarrow{f} 10$$

$$8 \xrightarrow{g} 10 \xrightarrow{f} 10$$

$$f \circ g(x) = \{(r, 5), (4, 5), (4, 10), (8, 10)\}$$

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$$f \circ g(x) \rightarrow$$

$$1 \xrightarrow{g} 2 \xrightarrow{f} 1$$

$$3 \xrightarrow{g} 1 \xrightarrow{f} 2$$

$$a \xrightarrow{g} 2 \xrightarrow{f} 2$$

$$b \xrightarrow{g} 1 \xrightarrow{f} 2$$

$$g \circ f(x) \rightarrow$$

$$2 \xrightarrow{f} 1 \xrightarrow{g} 2$$

$$3 \xrightarrow{f} 2 \xrightarrow{g} 2$$

$$4 \xrightarrow{f} 5 \xrightarrow{g} 2$$

$$1 \xrightarrow{f} 2 \xrightarrow{g} 2$$

$$\Rightarrow f \circ g = \{(1, 1), (3, 2), (a, 2), (b, 2)\}$$

$$(z, r) \in f \circ g \Rightarrow \boxed{a = z}$$

$$\Rightarrow g \circ f = \{(r, 1), (r, 2), (f, 2), (1, 2)\}$$

$$(f, 1) \in g \circ f \Rightarrow$$

$$\Rightarrow f = 2 \Rightarrow \boxed{b = 5}$$

$$(a, b) = \boxed{(z, 5)}$$

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$$g(x) = ax + b \rightarrow g(rx + r) = a(rx + r) + b = \boxed{ra + b} = \boxed{r^3 - r}$$

$$\Rightarrow ra = r \rightarrow \boxed{a = \frac{r}{r}} \quad , \quad ra + b = -r \Rightarrow r\left(\frac{r}{r}\right) + b = -r \Rightarrow \boxed{b = -r - \frac{r}{r} = -\frac{1r}{r}}$$

$$\boxed{g(x) = \frac{r}{r}x - \frac{1r}{r}}$$

$$f(x) = cx + d \rightarrow f(f(x)) = f(cx + d) = c(cx + d) + d \Rightarrow \boxed{c^2x + cd + d} = \boxed{r^2x + r}$$

$$c^2 = r \rightarrow c = \pm r$$

$$cd + d = r \Rightarrow d(c + 1) = r \quad \begin{cases} c=r \Rightarrow d=1 \\ c=-r \Rightarrow d=-r \end{cases} \Rightarrow \boxed{f(x) = rx + 1} \quad \text{و} \quad \boxed{f(x) = -rx - r}$$

$$g \circ f(-1) = g(f(-1)) \Rightarrow \boxed{g(-1) = \frac{-r}{r} - \frac{1r}{r} = -\frac{1r}{r} = -1}$$

$$D_{g \circ f} \rightarrow x \in D_f \rightarrow x \in \mathbb{R}$$

$$f(x) \in D_g \rightarrow f(x) \in \mathbb{R} - \{0, \varepsilon\} \Rightarrow f(x) \neq 0, \varepsilon \rightarrow x \geq 0, x \neq r$$

$$f(x) = \begin{cases} \sqrt{rx} & x \geq 0 \\ 0 & x < 0 \end{cases} \Rightarrow D_f = \mathbb{R}$$

$$\Rightarrow \boxed{D_{g \circ f} = [0, +\infty) - \{r\}}$$

$$D_g = \frac{1}{x - rx} = \frac{1}{x(1-r)} \Rightarrow D_g = \mathbb{R} - \{0, \varepsilon\}$$

$$D_{(f+g) \circ f} \rightarrow x \in D_f \rightarrow \boxed{x \in [-1, 1]}$$

$$f(x) \in D_{f+g} \rightarrow f(x) \in [0, 1] \Rightarrow 0 \leq 1 - rx \leq 1 \rightarrow \begin{cases} 1 - rx \geq 0 \rightarrow rx \leq 1 \rightarrow x \leq \frac{1}{r} \\ 1 - rx \leq 1 \rightarrow -rx \leq 0 \rightarrow rx \geq 0 \rightarrow x \geq 0 \end{cases}$$

$$D_f = \{x \mid 1 - rx \geq 0, rx \leq 1\} \quad |x| \leq 1 \rightarrow \boxed{-1 \leq x \leq 1} \rightarrow D_f = \boxed{x \in [0, 1]} \cap \boxed{-1 \leq x \leq 1}$$

$$D_{f+g} = D_f \cap D_g \Rightarrow D_f = [-1, 1]$$

$$\left. \begin{array}{l} D_g = x \geq 0 \rightarrow [0, +\infty) \\ D_{f+g} = D_f \cap D_g \Rightarrow D_{f+g} = [-1, 1] \cap [0, +\infty) = [0, 1] \end{array} \right\} \cap \{x \in [0, 1]\} \rightarrow D_{f+g} = \boxed{[0, 1]} = D_{(f+g) \circ f}$$

$$f\left(\frac{rx+1}{x-r}\right) = \varepsilon x + \delta$$

(الف)

$$t = \frac{rx+1}{x-r} \rightarrow x = \frac{-rt-1}{t-r}$$

$$\Rightarrow x = \frac{rt+1}{t-r} \Rightarrow \text{با جایگزینی می بینیم}$$

$$f\left(\frac{rt+1}{t-r}\right) + \delta = \frac{rt + \varepsilon + \delta t - 1}{t-r}$$

$$\frac{1rt-1}{t-r} \rightarrow \boxed{f(x) = \frac{1rx-1}{x-r}}$$

$$f\left(x + \frac{1}{x}\right) = x^r + \frac{1}{x^r}$$

(ب)

$$x + \frac{1}{x} = t \quad \text{ضرب کنیم} \quad x^r + \frac{1}{x^r} + rx + \frac{r}{x} = t^r$$

$$\Rightarrow t^r = x^r + \frac{1}{x^r} + r\left(x + \frac{1}{x}\right) \Rightarrow t^r = x^r + \frac{1}{x^r} + rt$$

$$\Rightarrow t^r - rt = x^r + \frac{1}{x^r}$$

$$\Rightarrow f(t) = t^r - rt \rightarrow \boxed{f(x) = x^r - rx}$$

$$g(f(x)) = 0 \rightarrow \text{هرجا برابر صفر نشود یعنی محورها را قطع کرده}$$

$$g(x) = 0 \rightarrow \text{در این نقاط تابع } g \circ f \text{ صفر می شود. یعنی در این نقاط محورها را قطع می کند}$$

$$x = 1, \sqrt{r} \Rightarrow f(x) = 1 \rightarrow x\sqrt{x} = 1 \quad \boxed{x = 1}$$

$$r-1 = \boxed{1}$$

$$f(x) = \sqrt{r} \rightarrow x\sqrt{x} = \sqrt{r} \rightarrow \boxed{x = r}$$