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نام و نام خانوادگی: امیرجبین خاکی پاسخنامه تشریحی تکلیف شماره ۹ کلاس: یازدهم ۸

$$f(f(\frac{r}{r})) = ?$$

$$f(\frac{r}{r}) = 6 + \frac{\pi}{r} \times \frac{r}{r} = 6 + \frac{\pi}{r} = \sqrt{r}$$

$$\Rightarrow f(\frac{r}{r}) = \sqrt{r}$$

$$f \circ f(\frac{r}{r}) = f(f(\frac{r}{r})) = f(\sqrt{r}) = \sqrt{(\sqrt{r})^2 + 1} = \boxed{2} \checkmark$$

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$$f(g(\sqrt{r})) = ?$$

$$g(\sqrt{r}) = \frac{\sqrt{r}}{1-\sqrt{r}} \approx \frac{1.4}{1-1.4} = \frac{1.4}{-0.4} = -3.5$$

$$f(-3.5) = [-3.5] = \boxed{-5} \checkmark$$

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$$f \circ g(\frac{\pi}{r}) = f(g(\frac{\pi}{r})) = ?$$

$$g(\frac{\pi}{r}) = 2 \times \cos \frac{\pi}{r} \quad \cos \frac{\pi}{r} = \frac{1}{r} \quad 2 \times \frac{1}{r} = \frac{2}{r}$$

$$f(g(\frac{\pi}{r})) = f(\frac{2}{r}) \Rightarrow \frac{1}{x} = \frac{1}{r} \Rightarrow x = r$$

$$f(\frac{2}{r}) = \sqrt{\frac{r(r)-1}{r^2}} = \sqrt{\frac{r}{r}} = \boxed{\frac{\sqrt{r}}{r}} \checkmark$$

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$$g \circ f(\frac{\pi}{r}) = g(f(\frac{\pi}{r})) = ?$$

$$f(\frac{\pi}{r}) = \sin \frac{\pi}{r} = \frac{\sqrt{r}}{r}$$

$$\Rightarrow g(\frac{\sqrt{r}}{r}) = \frac{\sqrt{r}}{r} \sqrt{1 - (\frac{\sqrt{r}}{r})^2} = \frac{\sqrt{r}}{r} \sqrt{1 - \frac{1}{r}} = \frac{\sqrt{r}}{r} \sqrt{\frac{r-1}{r}} = \frac{\sqrt{r}}{r} \times \frac{1}{\sqrt{r}} = \boxed{\frac{1}{r}} \checkmark$$

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$$r \xrightarrow{g} 4 \xrightarrow{g} 8$$

$$2 \xrightarrow{g} 4 \xrightarrow{g} 4$$

$$4 \xrightarrow{g} 8 \xrightarrow{g} 10$$

$$8 \xrightarrow{g} 10 \xrightarrow{g} x$$

$$g \circ g(x) = \{(r, 8), (4, 4), (4, 10)\} \checkmark$$

$$r \xrightarrow{f} 5 \xrightarrow{f} x$$

$$4 \xrightarrow{f} 5 \xrightarrow{f} x$$

$$8 \xrightarrow{f} 10 \xrightarrow{f} x$$

$$10 \xrightarrow{f} 10 \xrightarrow{f} x$$

$$f \circ f(x) = \emptyset \checkmark$$

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$$f \circ g(x) = \{(r, 5), (4, 5), (4, 10), (8, 10)\} \checkmark$$

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$$f \circ g(x) \rightarrow \begin{matrix} 1 \xrightarrow{g} 2 \xrightarrow{f} 1 \\ 3 \xrightarrow{g} 1 \xrightarrow{f} 3 \\ 4 \xrightarrow{g} 2 \xrightarrow{f} 2 \\ 5 \xrightarrow{g} 1 \xrightarrow{f} 3 \end{matrix}$$

$$\Rightarrow f \circ g = \{(1, 1), (3, 3), (4, 2), (5, 3)\}$$

$$(x, r) \in f \circ g \Rightarrow \boxed{a = x} \checkmark$$

$$g \circ f(x) \rightarrow \begin{matrix} 1 \xrightarrow{f} 2 \xrightarrow{g} 2 \\ 3 \xrightarrow{f} 1 \xrightarrow{g} 3 \\ 4 \xrightarrow{f} 2 \xrightarrow{g} 2 \\ 5 \xrightarrow{f} 1 \xrightarrow{g} 3 \end{matrix}$$

$$\Rightarrow g \circ f = \{(1, 2), (3, 3), (4, 2), (5, 3)\}$$

$$(f, 1) \in g \circ f \Rightarrow \boxed{b = 5} \checkmark$$

$$(a, b) = \boxed{(x, 5)} \checkmark$$

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$$g(x) = ax + b \rightarrow g(rx + r) = a(rx + r) + b = \boxed{ra + b} = \boxed{r^3 - r}$$

$$\Rightarrow ra = r \rightarrow \boxed{a = \frac{r}{r}} \quad , \quad ra + b = -r \Rightarrow r\left(\frac{r}{r}\right) + b = -r \Rightarrow \boxed{b = -r - \frac{r}{r} = -\frac{1r}{r}} \quad (2)$$

$$\boxed{g(x) = \frac{r}{r}x - \frac{1r}{r}}$$

$$f(x) = cx + d \rightarrow f(f(x)) = f(cx + d) = c(cx + d) + d \Rightarrow \boxed{c^2x + cd + d} = \boxed{r^2x + r}$$

$$c^2 = r \rightarrow c = \pm \sqrt{r}$$

$$cd + d = r \Rightarrow d(c + 1) = r \quad \begin{cases} c = \sqrt{r} & d = 1 \\ c = -\sqrt{r} & d = -r \end{cases} \Rightarrow \boxed{f(x) = rx + 1} \quad \text{یا} \quad \boxed{-rx - r}$$

$$g \circ f(-1) = g(f(-1)) \Rightarrow g(-1) = \frac{r}{r}(-1) - \frac{1r}{r} = -\frac{1r}{r} = -1 \quad (-1)$$

$$D_{g \circ f} \rightarrow x \in D_f \rightarrow x \in \mathbb{R}$$

$$f(x) \in D_g \rightarrow f(x) \in \mathbb{R} - \{0, \varepsilon\} \Rightarrow f(x) \neq 0, \varepsilon \rightarrow x > 0, x \neq \frac{1}{\varepsilon}$$

$$f(x) = \begin{cases} \sqrt{rx} & x \geq 0 \\ 0 & x < 0 \end{cases} \Rightarrow D_f = \mathbb{R}^+ \checkmark$$

$$\Rightarrow D_{g \circ f} = (0, +\infty) - \left\{ \frac{1}{\varepsilon} \right\}$$

$$D_g = \frac{1}{x^2 - rx} = \frac{1}{x(x - r)} \Rightarrow D_g = \mathbb{R} - \{0, \varepsilon\} \checkmark$$

$$D_{(f+g) \circ f} \rightarrow x \in D_f \rightarrow \boxed{x \in [-1, 1]}$$

$$f(x) \in D_{f+g} \rightarrow f(x) \in [0, 1] \Rightarrow 0 \leq 1 - x^2 \leq 1 \rightarrow \begin{cases} 1 - x^2 \geq 0 \rightarrow x^2 \leq 1 \rightarrow -1 \leq x \leq 1 \\ 1 - x^2 \leq 1 \rightarrow -x^2 \leq 0 \rightarrow x^2 \geq 0 \rightarrow x \in \mathbb{R} \end{cases}$$

$$D_f = \{x \mid 1 - x^2 \geq 0, x^2 \leq 1\} \rightarrow \boxed{-1 \leq x \leq 1} \rightarrow D_f = \boxed{x \in [-1, 1]} \quad (1, 2, 5)$$

$$D_{f+g} = D_f \cap D_g \Rightarrow D_f = [-1, 1] \quad D_g = x > 0 \rightarrow [0, +\infty) \Rightarrow D_{f+g} = [-1, 1] \cap [0, +\infty) = [0, 1]$$

$$D = \{x \in D_f, f(x) \in D_{f+g}\} \rightarrow 0 \leq f(x) \leq 1 \rightarrow 0 \leq \sqrt{1-x^2} \leq 1 \rightarrow \boxed{x \in [-1, 1]}$$

$$f\left(\frac{rx+1}{x-r}\right) = \varepsilon x + d$$

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$$t = \frac{rx+1}{x-r} \rightarrow x = \frac{-rt-1}{t-r}$$

$$\Rightarrow x = \frac{rt+1}{t-r} \Rightarrow \text{یا جایگزینی کنیم}$$

$$f\left(\frac{rt+1}{t-r}\right) + d = \frac{rt+1}{t-r} + d = \frac{rt+1+d(t-r)}{t-r}$$

$$\frac{rt+1}{t-r} \rightarrow \boxed{f(x) = \frac{rx-1}{x-r}} \checkmark$$

$$f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$$

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$$x + \frac{1}{x} = t \quad \text{یا} \quad x^2 + \frac{1}{x^2} + rx + \frac{r}{x} = t^2$$

$$\Rightarrow t^2 = x^2 + \frac{1}{x^2} + r\left(x + \frac{1}{x}\right) \Rightarrow t^2 = x^2 + \frac{1}{x^2} + rt$$

$$\Rightarrow t^2 - rt = x^2 + \frac{1}{x^2} \Rightarrow \boxed{f(t) = t^2 - rt} \checkmark$$

$$\Rightarrow \boxed{f(x) = x^2 - rx} \checkmark$$

هرجا برابر صفر نشود یعنی صفرها را قطع کرده

در این نقاط تابع $g \circ f$ صفر می شود. یعنی در این نقاط مخرج صفر می شود و قطع می شود

$$a = 1, 2\sqrt{2} \Rightarrow f(x) = 1 \rightarrow x\sqrt{x} = 1 \rightarrow \boxed{x = 1} \checkmark$$

$$2-1 = 1 \checkmark$$

$$f(x) = 2\sqrt{2} \rightarrow x\sqrt{x} = 2\sqrt{2} \rightarrow \boxed{x = 2} \checkmark$$