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نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

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$$f(x)\begin{cases}\cot\frac{x\pi}{4} & x\le 1\\ \sqrt{x^2+1} & x>1\end{cases}$$

$$\to f\left(\frac{2}{3}\right)=\cot\left(\frac{\frac{2\pi}{3}}{4}\right)=\cot\left(\frac{2\pi}{12}\right)=\cot\frac{\pi}{6}$$

$$\to \frac{\cos\left(\frac{\pi}{6}\right)}{\sin\left(\frac{\pi}{6}\right)}=\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}=\sqrt{3}$$

$$\to f(\sqrt{3})=\sqrt{(\sqrt{3})^2+1}=\sqrt{3+1}=\sqrt{4}=2$$

پاسخ صحیح ✓

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الف) $f\left(\frac{1}{x}\right)=\sqrt{\frac{2x-1}{x^2}}$ ، $g(x)=2\cos^2 x \to fog\left(\frac{\pi}{3}\right)$

$$\to g\left(\frac{\pi}{6}\right)=g(30^\circ)=2\cos\left(\frac{1}{2}\right)^2=2\times\left(\frac{1}{2}\right)^2=\frac{1}{2}$$

$$\to f\left(\frac{1}{x}\right)=f\left(\frac{1}{2}\right)=\sqrt{\frac{4-1}{4}}=\sqrt{\frac{3}{4}}=\boxed{\frac{\sqrt{3}}{2}}$$ ✓

ب) $f(x)=[x]$ ، $g(x)=\frac{x}{1-x}$

$$\to \frac{\sqrt{2}}{1-\sqrt{2}}\times\frac{1+\sqrt{2}}{1+\sqrt{2}}=\frac{\sqrt{2}+2}{1-2}=-\sqrt{2}-2$$

$$\to [-\sqrt{2}-2]=\boxed{-4}$$ ✓

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$f(x)=\sin x$ ، $g(x)=x\sqrt{1-x^2}$ $\to gof\left(\frac{\pi}{4}\right)\to$

$$f(x)=f\left(\frac{\pi}{4}\right)=f(45^\circ)=\sin 45^\circ=\frac{\sqrt{2}}{2}$$

$$\to g(x)=g\left(\frac{\sqrt{2}}{2}\right)=\frac{\sqrt{2}}{2}\sqrt{1-\left(\frac{\sqrt{2}}{2}\right)^2}=\frac{\sqrt{2}}{2}\sqrt{1-\frac{1}{2}}=\sqrt{\frac{2}{4}}\times\frac{\sqrt{2}}{2}=\frac{2}{4}=\boxed{\frac{1}{2}}$$ ✓

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الف) $fog(x)=\{(4,5),(6,12),(2,5),(8,12)\}$ ✓

ب) $gof(x)=\emptyset$ ✓

ج) $fof(x)=\emptyset$ ✓

د) $gog(x)=\{(4,8),(2,6),(6,10)\}$ ✓

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$f=\{(2,1),(3,2),(4,5),(1,7)\}$ $g=\{(1,2),(3,1),(a,3),(b,1)\}$

$(4,2)\in fog \to f(g(4))=2$ ، $a=4\to g(4)=3\to \boxed{a=4}$ ✓

$(4,1)\in gof\to g(5)=1\to(5,1)\to \boxed{b=5}$ ✓

$\to (a,b)=(4,5)$ ✓

پاسخ صحیح ✓

$f(f(x)) = 4x+3$ ۰ $f(-1) = -1$

$g(2x+3) = 2x-2 \rightarrow a(ax+b)+b = 4x+3 \rightarrow a^2x+ab+b = 4x+3$

$4x+3 \rightarrow a^2=4 \rightarrow a=\pm 2 \begin{cases} 4a+2b=4x+3 \rightarrow \boxed{b=1} \\ 4a-b=4x+3 \rightarrow b=-3 \end{cases}$

$f(x) = 2x+1 = 2(-1)+1 = -1 \rightarrow f(-1) = -1$

$f(a) = -2a-2 = -2(-1)-2 = -1 \rightarrow 2a+2-1 \rightarrow 2a=-4 \rightarrow \boxed{a=-2}$ ✓

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$f(x) = \sqrt{x+|x|}$ ، $g(x) = \frac{1}{x^2-4x}$

$\sqrt{x+|x|} = \begin{cases} x \geq 0 & \sqrt{x+x} = \sqrt{2x} \\ x<0 & \sqrt{x-x} = 0 \end{cases}$

$\rightarrow g \circ f = \rightarrow \frac{1}{(x+|x|)-4\sqrt{x+|x|}} \Rightarrow$ ریشه $\rightarrow R^+ - \{4\}$

$\Rightarrow D_f = R^+ - \{8\}$ ✓

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$f(x) = \sqrt{1-x^2}$ $\quad D_f = 1-x^2 \geq 0$ ✓

$g(x) = \sqrt{x}$ $\quad D_g = x \geq 0$ ✓

$\rightarrow \sqrt{\sqrt{1-x^2}} \rightarrow \sqrt{1-x^2} \geq 0$

$(f+g) \circ f \rightarrow (f+g)(f(x)) = f(f(x)) + g(f(x))$

$\xrightarrow{\text{ریشه}} [-1,1]$ ✓ $\quad \hookrightarrow f(\sqrt{1-x^2}) = \sqrt{1-(\sqrt{1-x^2})^2} \rightarrow \sqrt{x^2} = |x|$

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$f\left(\frac{2x+1}{x-2}\right) = 4x+5 \rightarrow t = \frac{2x+1}{x-2} = 4x+5$

$t(x-2) = 2x+1 \rightarrow tx-2t = 2x+1 \rightarrow tx-2x = 2t+1$

$\rightarrow x(t-2) = 2t+1 \Rightarrow x = \frac{2t+1}{t-2} \Rightarrow f(x) = \boxed{\frac{13x-11}{x-2}}$ ✓

$f\left(x+\frac{1}{x}\right) = x^3+\frac{1}{x^3} \Rightarrow x+\frac{1}{x} = t \Rightarrow f(t) = t^3-3t$

$\rightarrow f(x) = \boxed{x^3-3x}$ ✓

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$g(1) = 0$ ، $g(2\sqrt{2}) = 0$ $\quad f(x) = x\sqrt{x} \Rightarrow$ ریشه $= x \geq 0$

$g \circ f(x) = 0 \rightarrow f(x) = 1 \rightarrow x = 1 \rightarrow 2-1 = \boxed{1}$ ✓

$f(x) = 2\sqrt{2} \rightarrow x = 8$

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