

۲۵ عالی فرست

$$\Rightarrow \frac{2}{3} < 1 \Rightarrow f\left(\frac{2}{3}\right) = \cot\left(\frac{\pi \times \frac{2}{3}}{4}\right) = \cot \frac{\pi}{6} = \sqrt{3}$$

$$\Rightarrow \sqrt{3} > 1 \Rightarrow f(\sqrt{3}) = \sqrt{(\sqrt{3})^2 + 1} = \sqrt{4} = 2 \quad \checkmark$$

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$$\Rightarrow g\left(\frac{\pi}{3}\right) = 2 \cos^2\left(\frac{\pi}{3}\right) = 2\left(\frac{1}{2}\right)^2 = \frac{1}{2} \Rightarrow f\left(\frac{1}{2}\right) \rightarrow f\left(\frac{1}{2}\right)$$

$$\Rightarrow a = 2 \Rightarrow f\left(\frac{1}{2}\right) = \sqrt{\frac{2 \times 2 - 1}{2}} = \sqrt{\frac{3}{2}} = \frac{\sqrt{6}}{2} \quad \checkmark$$

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$$\Rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1 - \sqrt{2}} \times \frac{1 + \sqrt{2}}{1 + \sqrt{2}} = \frac{\sqrt{2}(1 + \sqrt{2})}{-1} = -2 - \sqrt{2}$$

$$f \circ g(\sqrt{2}) = [-2 - \sqrt{2}] \rightarrow \sqrt{2} \approx 1.4 \Rightarrow -2 - \sqrt{2} \approx -3.4 \rightarrow [-3.4] = -4 \quad \checkmark$$

$$\Rightarrow f\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \Rightarrow$$

$$\Rightarrow g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \times \sqrt{1 - \left(\frac{\sqrt{2}}{2}\right)^2} = \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2} \quad \checkmark$$

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$$f \rightarrow \Delta \rightarrow X \quad g \rightarrow \Delta \rightarrow X$$

$$A \rightarrow 12 \rightarrow X \quad 10 \rightarrow 12 \rightarrow X$$

$$g \circ f = \emptyset \quad \checkmark$$

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$$f \rightarrow g \rightarrow \Delta \quad 2 \rightarrow f \rightarrow \Delta$$

$$6 \rightarrow A \rightarrow 12 \quad A \rightarrow 10 \rightarrow 12$$

$$f \circ g = \{(4, 5), (2, 5), (4, 12), (8, 12)\} \quad \checkmark$$

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$$f \rightarrow g \rightarrow A \quad 2 \rightarrow f \rightarrow g$$

$$4 \rightarrow A \rightarrow 10 \quad A \rightarrow 10 \rightarrow X$$

$$g \circ g = \{(4, 1), (2, 4), (4, 10)\} \quad \checkmark$$

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$$f \circ f = \emptyset \quad \checkmark$$

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$$(4, 2) \Rightarrow f(g(4)) = 2 \Rightarrow g(4) = 2 \rightarrow f(2) = 2 \Rightarrow a = 4 \quad \checkmark$$

$$(4, 1) \Rightarrow g(f(4)) = 1 \Rightarrow f(4) = 5 \rightarrow g(5) = 1 \Rightarrow b = 5 \quad \checkmark$$

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$$(4, 5) \quad \checkmark$$

$f(x) = ax + b \Rightarrow f(f(x)) = a^2x + ab + b = 4x + 3 \Rightarrow a = 2, b = -1$ $a = 2 \Rightarrow 2b + b = 3 \Rightarrow b = 1 \xrightarrow{a=-1} -2b + b = 3 \Rightarrow b = -3$ $g(2x+3) = 3x-1 \xrightarrow{\tilde{1}} t = 2x+3 \Rightarrow x = \frac{t-3}{2} \Rightarrow g(t) = 3\left(\frac{t-3}{2}\right) - 1$ $= \frac{3t-11}{2} \Rightarrow g(x) = \frac{3x-11}{2}$ $a=2 \Rightarrow g \circ f(-1) = f(-1) = (-2+1) = -1 \Rightarrow g(-1) = \frac{-3-11}{2} = -7$ $a=-1 \Rightarrow g \circ f(-1) = f(-1) = -3+2 = -1 \Rightarrow g(-1) = -7$	2 6
$\Rightarrow x + x > 0 \Rightarrow x \leq 0 \Rightarrow f(x) = 0$ $\Rightarrow x > 0 \Rightarrow f(x) = \sqrt{2x}$ $\Rightarrow x^2 - 4x \neq 0 \Rightarrow x(x-4) = 0 \Rightarrow x \neq 0 \leq 4 \Rightarrow f(x) \neq 4, 0$ $f(x) \neq 0 \Rightarrow x > 0$ ① $f(x) \neq 4 \Rightarrow \sqrt{2x} = 4 \Rightarrow 2x = 16 \Rightarrow x = 8 \Rightarrow D_{g \circ f} = (0, +\infty) - \{8\}$	2 7
$(f+g) \circ f \rightarrow ((f+g) \circ f)(x) = f(f(x)) + g(f(x))$ $D_f = \{1 - x^2 \geq 0\} \Rightarrow -1 \leq x \leq 1 \Rightarrow D_f = [-1, 1]$ ① $g(f(x)) = \sqrt{1-x^2} \Rightarrow \sqrt{1-x^2} \geq 0 \Rightarrow -1 \leq x \leq 1$ برقرار است ② $f(f(x)) = \sqrt{1-(\sqrt{1-x^2})^2} = \sqrt{x^2} = x \rightarrow x \in [-1, 1]$ $\xrightarrow{①, ②} D_{(f+g) \circ f} = [-1, 1]$	2 8
$\xrightarrow{\tilde{1}} y = \frac{3x+1}{x-3} \Rightarrow y(x-3) = 3x+1 \Rightarrow yx - 3y = 3x+1 \Rightarrow x = \frac{3y+1}{y-3}$ الف $f(y) = 4x+5 \Rightarrow \frac{12y+4}{y-3} + 5 = \frac{12y+4+5y-15}{y-3} \Rightarrow f(x) = \frac{17x-11}{x-3}$	2
$\Rightarrow t = x + \frac{1}{x} \Rightarrow x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$ $\Rightarrow f(t) = t^3 - 3t \Rightarrow f(x) = x^3 - 3x$	9
$g(x) = 0 \Rightarrow x = 1 \leq 2\sqrt{2} \quad f(x) = x\sqrt{x} = x^{\frac{3}{2}}$ $g \circ f_{\text{max}} = 0 \Rightarrow f(x) = 1 \leq 2\sqrt{2} \xrightarrow{1} f(x) = 1 \Rightarrow x^{\frac{3}{2}} = 1 \Rightarrow x = 1^{\frac{2}{3}} = 1$ $\xrightarrow{2\sqrt{2}} f(x) = 2\sqrt{2} \Rightarrow x^{\frac{3}{2}} = 2\sqrt{2} \Rightarrow x = \sqrt[3]{2}$ $x_2 - x_1 \Rightarrow 2 - 1 = 1$	2