

$$(10) f(x) + g(x) = r\sqrt{r+\sqrt{1-x}} \Rightarrow r\sqrt{r+\sqrt{1-x}} \cdot \sqrt{r+1} \Rightarrow r\sqrt{(r+1)+\sqrt{1-x}}$$

$$f(x) + g(x) = r\sqrt{r+1} + r\sqrt{1-x} \quad ; \quad f(x) = r\sqrt{r+1} + r\sqrt{1-x}$$

$$g(x) = (r\sqrt{r+1} + r\sqrt{1-x}) - (r\sqrt{r+1} + r\sqrt{1-x}) = \sqrt{r+1} - \sqrt{1-x}$$

$$y = \frac{f(x)}{r} - \frac{g(x)}{r} \Rightarrow \frac{r\sqrt{r+1} + r\sqrt{1-x}}{r} - \frac{\sqrt{r+1} - \sqrt{1-x}}{r}$$

$$\Rightarrow y = \frac{\sqrt{r+1} + r\sqrt{1-x} - \sqrt{r+1} + \sqrt{1-x}}{r} = \frac{r\sqrt{1-x} + \sqrt{1-x}}{r} = \sqrt{1-x}$$

$$D_f = D_g = [-1, 1], \quad R = [0, \sqrt{r}]$$

$$\Rightarrow y = \sqrt{1-x}, \quad D = [-1, 1], \quad R = [0, \sqrt{r}]$$

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$$(7) f(x) = \begin{cases} \cot \frac{\pi x}{r} & ; x \leq 1 \\ \sqrt{x^2+1} & ; x > 1 \end{cases}$$

$$f \circ f\left(\frac{r}{r}\right) = ?$$

$$f \circ f\left(\frac{r}{r}\right) = f\left(f\left(\frac{r}{r}\right)\right)$$

$$\Rightarrow f\left(\frac{r}{r}\right) = \cot \frac{\frac{r}{r}\pi}{r} = \cot \frac{1 \cdot \pi}{r} = \cot \frac{\pi}{r} = \sqrt{r}, \quad f(\sqrt{r}) = \sqrt{(\sqrt{r})^2+1} = \sqrt{r+1} = r \Rightarrow f \circ f\left(\frac{r}{r}\right) = r$$

$$(2) f\left(\frac{1}{x}\right) = \sqrt{\frac{r x - 1}{x^2}}, \quad g(x) = r \cos^2 x, \quad f \circ g\left(\frac{\pi}{r}\right) = ?$$

$$f \circ g\left(\frac{\pi}{r}\right) = f\left(g\left(\frac{\pi}{r}\right)\right) \Rightarrow g\left(\frac{\pi}{r}\right) = r \cos^2\left(\frac{\pi}{r}\right) = r \left(\frac{1}{r}\right)^2 = \frac{1}{r}$$

$$f\left(\frac{1}{r}\right) = \sqrt{\frac{r\left(\frac{1}{r}\right) - 1}{\left(\frac{1}{r}\right)^2}} = \sqrt{\frac{r}{r}} = \frac{\sqrt{r}}{r} = f \circ g\left(\frac{\pi}{r}\right)$$

② ب) $f(x) = [x]$, $g(x) = \frac{x}{1-x}$, $f \circ g(x) = ?$

$$f \circ g(x) = f(g(x)) \Rightarrow g(x) = \frac{x}{1-x} \times \frac{1+x}{1+x} = \frac{x+x^2}{1-x^2} = -(\sqrt{x}+x)$$

$$\Rightarrow f \circ g(x) = [-(\sqrt{x}+x)] = [-(1, \sqrt{x}+x)] = [-(x, \sqrt{x}+x)] = -x$$

$$\Rightarrow \boxed{f \circ g(x) = -x}$$

③ $f(x) = \sin x$, $g(x) = x\sqrt{1-x^2}$, $g \circ f(\frac{\pi}{4}) = ?$

$$g \circ f(\frac{\pi}{4}) = g(f(\frac{\pi}{4})) \Rightarrow f(\frac{\pi}{4}) = \sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}, g(\frac{\sqrt{2}}{2})$$

$$= \frac{\sqrt{2}}{2} \times \sqrt{1 - (\frac{\sqrt{2}}{2})^2} \Rightarrow g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2\sqrt{2}} = \frac{1}{2} = g \circ f(\frac{\pi}{4})$$

④ $g(x) = \{(t, q), (r, e), (s, 1), (1, 10)\}$

$$f(x) = \{(t, \Delta), (q, \Delta), (1, 12), (1, 12)\}$$

ا) $f \circ g(x) = f(g(x)) \Rightarrow$

$$\begin{array}{l} t \rightarrow q \rightarrow \Delta \\ r \rightarrow t \rightarrow \Delta \\ s \rightarrow 1 \rightarrow 12 \\ 1 \rightarrow 10 \rightarrow 12 \end{array} = \{(t, \Delta), (r, \Delta), (s, 12), (1, 12)\}$$

ب) $g \circ f(x) = g(f(x)) \Rightarrow$

$$\begin{array}{l} t \rightarrow \Delta \rightarrow x \\ q \rightarrow \Delta \rightarrow x \\ 1 \rightarrow 12 \rightarrow x \\ 10 \rightarrow 12 \rightarrow x \end{array} \Rightarrow g \circ f = \emptyset$$

ج) $f \circ f(x) = f(f(x))$

$$\begin{array}{l} t \rightarrow \Delta \rightarrow x \\ q \rightarrow \Delta \rightarrow x \\ 1 \rightarrow 12 \rightarrow x \\ 10 \rightarrow 12 \rightarrow x \end{array} \Rightarrow f \circ f = \emptyset$$

د) $g \circ g(x) = g(g(x))$

$$\begin{array}{l} t \rightarrow q \rightarrow 1 \\ r \rightarrow t \rightarrow q \\ s \rightarrow 1 \rightarrow 10 \\ 1 \rightarrow 10 \rightarrow x \end{array} \Rightarrow g \circ g = \{(s, 1), (r, q), (s, 10)\}$$

$$(5) f: \{(1,1), (1,2), (1,3), (1,4)\}, g: \{(1,2), (1,1), (2,1), (3,1)\}$$

$$(1,2) \in f \circ g, (1,1) \in g \circ f, (a,b) = ?$$

$$g(1) = 2, f(2) = 1 \Rightarrow y = 2, x = 1, f(1) = 3, g(3) = 1 \Rightarrow b = 3$$

$$\Rightarrow (a,b) = (1,3)$$

$$(6) f, g \rightarrow \text{تابعی}, f(f(n)) = f(n+1), g(f(n+1)) = f(n-1)$$

$$g \circ f(-1) = ?$$

$$f(n) = an + b \Rightarrow f \circ f(n) = a(an + b) + b$$

$$\Rightarrow f \circ f(n) = a^2 n + ab + b$$

$$f \circ f(n) = f(n+1)$$

$$a^2 = 1 \Rightarrow a = \pm 1$$

$$ab + b = 1$$

$$\Rightarrow a \begin{cases} +1 \rightarrow f(n) = n+1 = -1 \\ -1 \rightarrow f(n) = -n-1 = -1 \end{cases}$$

$$\Rightarrow g(-1) = -1 \Rightarrow g \circ f(-1) = g(-1) = -1$$

$$(7) f(n) = \sqrt{n+|n|}, g(n) = \frac{1}{n^2 - f(n)}, D(g \circ f) = ?$$

$$f(n) = \sqrt{n+|n|} \Rightarrow f(n) = \begin{cases} 0; n \leq 0 \\ \sqrt{2n}; n > 0 \end{cases} \Rightarrow D_f = \mathbb{R}$$

$$g(n) = \frac{1}{n^2 - f(n)} = \frac{1}{n(n-f)} \Rightarrow f(n) \neq 0, f(n) \neq 1 \Rightarrow f(n) = 0 \Rightarrow n \leq 0$$

$$\Rightarrow f(n) = 1 \Rightarrow \sqrt{n+|n|} = 1 \Rightarrow n+|n| = 1 \Rightarrow 4n \geq 1 \Rightarrow n \geq 1$$

$$\Rightarrow D(g \circ f) = (0, 1) \cup (1, +\infty) = (0, +\infty) - \{1\}$$

$$(8) f(n) = \sqrt{1-n^2}, g(n) = \sqrt{n}, D((f+g) \circ f) = ?$$

$$(f+g) \circ f = f(f(n)) + g(f(n)) \Rightarrow f(n) = \sqrt{1-n^2} \Rightarrow 1-n^2 \geq 0 \Rightarrow -1 \leq n \leq 1$$

$$\Rightarrow D_f = [-1, 1], D_{(f+g) \circ f} = [-1, 1] \leftarrow \text{but } f \text{ is not defined for } n < -1 \text{ or } n > 1$$

9 الف) $f\left(\frac{r_{n+1}}{n-r}\right) = f_{n+1} \Rightarrow \frac{r_{n+1}}{n-r} = t \Rightarrow \frac{rt+1}{t-r} = n$

$$f(t) = t\left(\frac{rt+1}{t-r}\right) + 1 \Rightarrow f(t) = \frac{rt+1}{t-r} + \frac{rt-1}{t-r} = \frac{1rt-1}{t-r}$$

$$f(n) = \frac{1rn-1}{n-r}$$

ب) $f\left(n+\frac{1}{n}\right) = n^r + \frac{1}{n^r} \Rightarrow f\left(n+\frac{1}{n}\right) = \left(n+\frac{1}{n}\right)^r - r\left(n+\frac{1}{n}\right)$

$$\left(n+\frac{1}{n}\right)^r = n^r + \frac{1}{n^r} + r\left(n+\frac{1}{n}\right) \Rightarrow n^r + \frac{1}{n^r} = \left(n+\frac{1}{n}\right)^r - r\left(n+\frac{1}{n}\right)$$

$$\Rightarrow \boxed{f(n) = n^r - rn}$$

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$$g \circ f(n) = 0 \Rightarrow g(f(n)) = 0 \Rightarrow f(n) = \sqrt{r} \Rightarrow n\sqrt{n} = \sqrt{r} \Rightarrow n = r$$

$$f(n) = 1 \Rightarrow n\sqrt{n} = 1 \Rightarrow n = 1$$

$$\Rightarrow r, 1 = \text{نقطه میانی } g \circ f \Rightarrow \boxed{r-1=1}$$