نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

$\rightarrow f(\frac{\pi}{3}) = \cot \frac{\frac{1}{2}\pi}{\frac{\pi}{4}} = \cot \frac{\pi}{6} = \sqrt{3}$

$\rightarrow f(\sqrt{3}) = \sqrt{(\sqrt{3})^2+1} = \sqrt{4} = 2$ ۱

الف) $g(\frac{\pi}{3}) = 2\cos^2\frac{\pi}{3} = 2(\frac{1}{2})^2 = \frac{1}{2}$

$f(\frac{1}{2}) = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

ب) $g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}}$

$\rightarrow f(\frac{\sqrt{2}}{1-\sqrt{2}}) = [\frac{\sqrt{2}}{1-\sqrt{2}}] = [-3,4] = -4$ ۲

$f(\frac{\pi}{4}) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\rightarrow g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2}\sqrt{\underbrace{1-(\frac{\sqrt{2}}{2})^2}_{1-\frac{2}{4}=\frac{1}{2}}} = \sqrt{\frac{2}{4}} \times \sqrt{\frac{1}{2}} = \frac{1}{2}$ ۳

الف) $4 \rightarrow 6 \rightarrow 8$ $\quad 6 \rightarrow 8 \rightarrow 12$
$2 \rightarrow 4 \rightarrow 8$ $\quad 8 \rightarrow 10 \rightarrow 12$

ب) $4 \rightarrow 8 \rightarrow x$ $\quad 10 \rightarrow 12 \rightarrow x$
$6 \rightarrow 8 \rightarrow x$
$8 \rightarrow 12 \rightarrow x$

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د) $4 \rightarrow 6 \rightarrow 8$ $\quad 8 \rightarrow 10 \rightarrow x$
$2 \rightarrow 4 \rightarrow 6$
$6 \rightarrow 8 \rightarrow 10$ ۴

$(4,2) \in f\circ g$

$4 \rightarrow (3) \rightarrow 2$

$\downarrow$

$a=4$

$(4,1) \in g\circ f$

$4 \rightarrow 8 \rightarrow 1$

$\Rightarrow b=8$ ۵

$\rightarrow (a,b) = (4,8)$

اگر $f(x)=ax+b \Rightarrow f(f(x))=a(ax+b)+b=a^2x+ba+b$

$=4x+3$

$\Rightarrow \begin{cases} a^2=4 \rightarrow a=\pm 2 \\ ab+b=3 \end{cases}$ $\begin{cases} a=2 \rightarrow b=1 \\ a=-2 \rightarrow b=-3 \end{cases}$

$[g(2x+3)=3x-2]$

$2x+3=-1 \Rightarrow \boxed{x=-2}$

$f(x)=2x+1$ یا $f(x)=-2x-3$

$g\circ f(-1)=g(f(-1))=g(-1)=3(-2)-2=-8$

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$D_f \rightarrow g\circ f \rightarrow D_{g\circ f}=\{x\in D_f ; f(x)\in D_g\}$

$D_g=\mathbb{R}-\{0,4\}$

$D_f=\mathbb{R}$

$\sqrt{x+|x|}\neq 0 \rightarrow \boxed{x\neq 0, \mathbb{R}^-}$

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$\boxed{x>0}$

$\rightarrow D_{g\circ f}=(0,+\infty)-\{1\}$

$\sqrt{x+|x|}\neq 4$

$\rightarrow \boxed{x\neq 8}$

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$D_{(f+g)\circ f}=\{x\in D_f ; f(x)\in D_{(f+g)\circ f}\}$

$[-1,1]$ $\quad$ $[0,1]$

$\boxed{D_f=[-1,1]}$

$0\leq \sqrt{1-x^2}\leq 1$

$0\leq x^2\leq 1 \leftarrow -1\leq -x^2\leq 0 \leftarrow 0\leq 1-x^2\leq 1$

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الف) $\frac{3x+1}{x-2}=t$

$\rightarrow 3x+1=tx-2t$

$x(3-t)=-(2t+1)$

$x=\frac{2t+1}{t-3} \rightarrow 4\left(\frac{2t+1}{t-3}\right)+a$

$\rightarrow f(x)=\frac{8x+4}{x-3}+a$

ب) $(x+\frac{1}{x})^3=x^3+\frac{1}{x^3}+3x+\frac{3}{x}$

$3(x+\frac{1}{x})$

$x+\frac{1}{x}=t \rightarrow (x+\frac{1}{x})^3=x^3+\frac{1}{x^3}+3(x+\frac{1}{x})$

$x^3+\frac{1}{x^3}=(x+\frac{1}{x})^3-3(x+\frac{1}{x})$

$x+\frac{1}{x}=t \rightarrow f(t)=t^3-3t$

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$g(1)=0$ , $g(2\sqrt{2})=0$ , $f(x)=x\sqrt{x}$

$g(x\sqrt{x})=0 \Rightarrow \begin{cases} x\sqrt{x}=1 \Rightarrow \boxed{x=1} \\ x\sqrt{x}=2\sqrt{2} \Rightarrow \boxed{x=2} \end{cases}$

اختلاف $\boxed{2-1=1}$