

$$f\left(\frac{2}{3}\right) = 0.5 \times \frac{2}{3} = 0.5 \times \frac{2}{3} = \frac{1}{3}$$

$$f \circ f\left(\frac{2}{3}\right) = f\left(f\left(\frac{2}{3}\right)\right) = f\left(\frac{1}{3}\right) = \frac{1}{3+1} = \frac{1}{4}$$

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$$\text{الف) } g\left(\frac{2}{3}\right) = 2 \cdot \log\left(\frac{2}{3}\right) = 2 \cdot \log\left(\frac{2}{3}\right) = 2 \times \frac{1}{2} = 1$$

$$\Rightarrow f \circ g\left(\frac{2}{3}\right) = f\left(g\left(\frac{2}{3}\right)\right) = f(1) = \frac{1}{1+1} = \frac{1}{2}$$

$$\text{ب) } g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \Rightarrow f \circ g(\sqrt{2}) = f\left(\frac{\sqrt{2}}{1-\sqrt{2}}\right) = \frac{1}{\frac{\sqrt{2}}{1-\sqrt{2}} + 1}$$

$$\frac{\sqrt{2}}{1-\sqrt{2}} = \frac{\sqrt{2}(1+\sqrt{2})}{1-\sqrt{2}(1+\sqrt{2})} = \frac{\sqrt{2} + 2}{1-2-\sqrt{2}} = \frac{\sqrt{2} + 2}{-1-\sqrt{2}} \Rightarrow f\left(\frac{\sqrt{2} + 2}{-1-\sqrt{2}}\right) = \frac{1}{\frac{\sqrt{2} + 2}{-1-\sqrt{2}} + 1} = \frac{1}{\frac{\sqrt{2} + 2 - 1 - \sqrt{2}}{-1-\sqrt{2}}} = \frac{1}{\frac{1}{-1-\sqrt{2}}} = -1 - \sqrt{2}$$

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$$f\left(\frac{2}{3}\right) = \log\left(\frac{2}{3}\right) = \frac{\sqrt{2}}{2}$$

$$g \circ f\left(\frac{2}{3}\right) = g\left(f\left(\frac{2}{3}\right)\right) = g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \times \sqrt{1 - \left(\frac{\sqrt{2}}{2}\right)^2} = \frac{\sqrt{2}}{2} \times \sqrt{\frac{1}{2}}$$

$$= \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$$

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$$\text{الف) } f \circ g(a) = \{(4, 5), (2, 5), (9, 12), (8, 12)\}$$

$$\text{ب) } g \circ f(a) \Rightarrow D_{g \circ f} = \{a \in D_f \mid f(a) \in D_g\} \Rightarrow D_{g \circ f} = \emptyset \Rightarrow g \circ f(a) = \emptyset$$

$$\text{ج) } f \circ f(a) \Rightarrow D_{f \circ f} = \{a \in D_f \mid f(a) \in D_f\} \Rightarrow D_{f \circ f} = \emptyset \Rightarrow f \circ f(a) = \emptyset$$

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$$\text{د) } g \circ g(a) = \{(4, 8), (2, 4), (9, 10)\}$$

$$f \circ g(4) = f(g(4)) = 2 \xrightarrow{f(3)=2} g(2) = 3 \Rightarrow (a, 3) = (4, 3) \Rightarrow a = 4$$

$$g \circ f(4) = g(f(4)) = 1 \xrightarrow{f(2)=1} g(1) = 1 \Rightarrow (b, 1) = (4, 1) \Rightarrow b = 4$$

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$$\Rightarrow (a, b) = (4, 4)$$

$$f(x) = ax + b \Rightarrow f(f(x)) : f(ax+b) = a(ax+b) + b = a^2x + ab + b$$

$$\rightarrow a^2x + ab + b = f(ax) = f(ax) \Rightarrow a^2 = f, b(a+1) = f \Rightarrow \begin{cases} a=f \Rightarrow b=1 \Rightarrow f(x) = f_{x+1} \\ a=-f \Rightarrow b=-f \Rightarrow f(x) = -f_{x-f} \end{cases}$$

$$g(x) = cx + d \Rightarrow g(f(x)) : g(ax+b) = c(ax+b) + d = f_{ax+b} = f_{ax} + f_{b+d} = f_{ax} + f$$

$$\Rightarrow f_{c, f} \Rightarrow c = \frac{f}{f} \quad f_{c+d} = -f \xrightarrow{c=\frac{f}{f}} f(\frac{f}{f}) + d = -f \Rightarrow d = -\frac{1f}{f} \Rightarrow g(x) = \frac{f}{f}x - \frac{1f}{f}$$

$$f_{(-1)} \left\langle \begin{matrix} f_{(x), f_{x+1}} \rightarrow f_{(-1)}, -1 \\ f_{(x), -f_{x-f}} \rightarrow f_{(-1)}, -1 \end{matrix} \right. \Rightarrow g(f_{(-1)}) : g(-1) = \frac{f}{f}(-1) - \frac{1f}{f} = -\frac{1f}{f} = \underline{-1}$$

$$g_{\text{set}} = \{x \in D_f \mid f(x) \in D_g\}$$

$$\Rightarrow D_f : x + |x| \geq 0 \Rightarrow x \geq -|x| \Rightarrow x \in \mathbb{R}$$

$$D_g : x^2 - f_{ax} \neq 0 \Rightarrow x(1-f) \neq 0 \Rightarrow x \neq 0, f$$

$$\Rightarrow f(x) \neq 0, f \Rightarrow \sqrt{x+|x|} \neq 0, f \Rightarrow x + |x| \neq 0, 1, f$$

$$\Rightarrow x > 0, x \neq 1 \Rightarrow D_{g \circ f} : (0, +\infty) - \{1\}$$

$$(f+g)_{\text{set}} = f_{\text{set}} \cup g_{\text{set}} \Rightarrow D_{f+g} : \{x \in D_f \mid f(x) \in D_g\}$$

$$\Rightarrow D_f : 1 \geq x^2 \Rightarrow [-1, 1], f(x) \in D_g \Rightarrow 1 \geq (\sqrt{1-x^2})^2 \Rightarrow 1 \geq 1-x^2 \Rightarrow x^2 \geq 0$$

$$\Rightarrow 0 \leq x^2 \leq 1 \Rightarrow \sqrt{x^2} \leq \sqrt{1} \Rightarrow D_{f+g} : [-1, 1]$$

$$D_{g \circ f} = \{x \in D_f \mid f(x) \in D_g\} \Rightarrow D_f : [-1, 1], f(x) \in D_g \Rightarrow \sqrt{1-x^2} \geq 0$$

$$\Rightarrow D_{g \circ f} : [-1, 1] \Rightarrow D_{f+g} \cap D_{g \circ f} = D_{(f+g)_{\text{set}}} = [-1, 1]$$

$$a) f\left(\frac{f_{x+1}}{x-f}\right) = f_{x+1} \rightarrow \frac{f_{x+1}}{x-f} = t \Rightarrow x = \frac{f_{t+1}}{t-f}$$

$$\Rightarrow f(t) : f\left(\frac{f_{t+1}}{t-f}\right) + a \Rightarrow f_{ax}, \frac{1_{ax+f} + a}{x-f}$$

$$b) f\left(x + \frac{1}{x}\right) : x^2 + \frac{1}{x^2} \rightarrow x + \frac{1}{x} = t \Rightarrow t^2 = x^2 + \frac{1}{x^2} + 2 \Rightarrow x^2 + \frac{1}{x^2} = t^2 - 2$$

$$= x^2 + \frac{1}{x^2} + 2 \Rightarrow x^2 + \frac{1}{x^2} = t^2 - 2 \Rightarrow f(t) : t^2 - 2 \Rightarrow f_{(x)} : x^2 - f_{ax}$$

$$g_{\text{set}}(x) : 0 \Rightarrow g(f(x)) : 0$$

$$\Rightarrow f(x) \left\langle \begin{matrix} 1 \Rightarrow x \sqrt{x} = 1 \Rightarrow x = 1 \\ \sqrt{x} \Rightarrow x \sqrt{x}, \sqrt{x} \Rightarrow x = 2 \end{matrix} \right.$$

$$\Rightarrow a_1, a_2 : 2-1, (1) \text{ سنج }$$