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اسمت پیدا نیست با ارفاق این بار رو در نظر می‌گیرم

$$f(x)=\begin{cases}\cot\frac{\pi x}{2} & ; x\le 1\\ \sqrt{x^2+1} & ; x>1\end{cases}$$

$f(\frac{\sqrt{3}}{2}) = \cot\frac{\pi}{6} \Rightarrow \frac{\pi}{6} = 30^\circ \Rightarrow \cot(30^\circ)=\sqrt{3}$ ۱-

$f(\sqrt{3}) = \sqrt{4} = \boxed{2}$

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الف) $f(\frac{1}{x}) = \sqrt{\frac{2x-1}{x^2}}$

$\frac{1}{x} = t \quad \frac{2x-1}{x^2} = \frac{2}{x} - \frac{1}{x^2} \rightarrow \frac{2x-1}{x^2} = 2t - t^2$

$f(t) = \sqrt{2t - t^2} \rightarrow f(x)=\sqrt{2x-x^2}$

$g(x) = 2\cos^2 x$

$\hookrightarrow g(\frac{\pi}{3}) = \frac{1}{2} \rightarrow f(\frac{1}{2}) = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$

ب) $f(x) = [x]$ و $g(x) = \frac{x}{1-x}$

$g(\sqrt{2}) = \frac{\sqrt{2}\times(1+\sqrt{2})}{1-\sqrt{2}\times(1+\sqrt{2})} = \frac{\sqrt{2}+2}{1^2-2} = -\sqrt{2}-2$

$\simeq -3,4$

$f(-\sqrt{2}-2) = [-\sqrt{2}-2] = \cancel{-3}\ -4$

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$f(x) = \sin x$, $g(x) = x\sqrt{1-x^2}$

$f(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} \rightarrow g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2}\times\sqrt{1-\frac{1}{2}} = \frac{\sqrt{2}}{2}\times\frac{1}{\sqrt{2}} = \boxed{\frac{1}{2}}$

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$g(x) = \{(4,6),(2,4),(6,8),(8,10)\}$

$f(x) = \{(4,5),(6,5),(8,12),(10,12)\}$

الف) $f\circ g(x) = \{(4,5),(2,5),(6,12),(8,12)\}$

ب) $g\circ f(x) = \varnothing$

ج) $f\circ f(x) = \varnothing$

د) $g\circ g(x) = \{(4,8),(2,6),(6,10)\}$

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$f = \{(2,1),(3,2),(4,5),(1,7)\}$ و $g = \{(1,2),(3,1),(a,3),(b,1)\}$

$(4,2)\in f\circ g \rightarrow g(4)=3 \rightarrow f(3) = 2 \longrightarrow \boxed{a=4}$

$(4,1)\in g\circ f \rightarrow f(4) = 5 \longrightarrow g(5) = 1 \longrightarrow \boxed{b=5}$

$(4,5)$

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$f(f(x)) = 4x+3$ $\quad ax+b$

$f(f(x)) = a(ax+b)+b$ $\quad a^2x+ab+b = 4x+3 \rightarrow \boxed{a=\pm 2}$

$f(f(x)) = a^2x+ab+b$

I) $a=+2 \rightarrow b=1 \rightarrow f(x) = +2x+1$

II) $a=-2 \rightarrow b=-3 \longrightarrow -2x-3$

$g(2x+3) = 3x-2$

$f(-1) = -1 \rightarrow 2x+3 = -1 \rightarrow \boxed{x=-2} \rightarrow g(-1) = \boxed{-8}$

ولی در هر صورت آنگه پیدا کنیس

۱- بگذاریم جواب منفی یک می‌شود

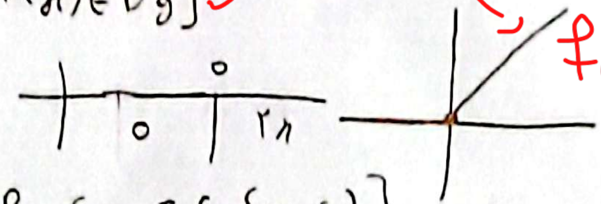
-۷ (۱)

$f(x)=\sqrt{x+|x|}\ ,\ g(x)=\dfrac{1}{x^2-4x}\to 0,4 \qquad D_{gof}=?$

$\to f(x)\neq 0 \to x>0$ (۱)

$f(x)\neq 4 \qquad x\neq 8$ (۲)

$D_{gof}=[x\in D_f\ ,\ f(x)\in D_g]$ ✓

$D_f:\ x+|x|\geq 0$ 

$D_f=R$ ✓ (۳)

$[x\in R\ ,\ f(x)\in f-\{0,4\}] \to D_{gof}=R-\{0,4\}$

$D_f=R-\{0,4\}$

$\to D_{gof}=(0,+\infty)-\{8\}$

$f(x)=\sqrt{1-x^2}\ ,\ g(x)\sqrt{x} \qquad D_{(f+g)of}=? \to D_{(f+g)of}=[-1,1]$ (۱)

$-1\leq x\leq 1 \qquad x\geq 0$

$D_{(f+g)of}=[x\in D_f\ ,\ f(x)\in D_{(f+g)}]$ ✓

$1-x^2\geq 0$

$1\geq x^2 \to 1\geq x\geq -1 = D_f$ (۱)

$[x\in[1,-1]\ ,\ f(x)\in D_{(f+g)}] \qquad D_{f+g}=D_f\cap D_g:\ 0\leq x\leq 1$

$\sqrt{1-x^2}+\sqrt{x}$

$0\leq f(x)\leq 1 \to x^2\geq 0$ ✓ برقرار (۲)

-۹ (۱,۷۵)

$f\left(\dfrac{3x+1}{x-2}\right)=4x+5 \qquad \dfrac{3x+1}{x-2}=a \qquad x=\dfrac{2a+1}{a-3}$ دقت!

$f(a)=4\left(\dfrac{2a+1}{a-3}\right)+5 \to f(a)=\dfrac{8a+4}{a-3}+5 \to \dfrac{8a+4+5a-15}{a-3}=\dfrac{13a-11}{a-3}$

$f(x)=\dfrac{13x-11}{x-3}$

$f\left(x+\dfrac{1}{x}\right)=x^3+\dfrac{1}{x^3} \qquad x+\dfrac{1}{x}=a \qquad f(a)=a^3-3a$

$\left(x+\dfrac{1}{x}\right)^3=x^3+\dfrac{1}{x^3}+3\left(x+\dfrac{1}{x}\right) \qquad f(x)=x^3-3x$ ✓

$x^3+\dfrac{1}{x^3}=\left(x+\dfrac{1}{x}\right)^3-3\left(x+\dfrac{1}{x}\right)$

a^3-3a

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$g \circ f(x) = 0$

$g = 0 \Rightarrow x = ۲\sqrt{۲} \quad \text{or} \quad x = ۱$

$x = ۲\sqrt{۲} \longrightarrow x\sqrt{x} = ۲\sqrt{۲} \longrightarrow x = ۲$ ✓

$x = ۱ \longrightarrow x\sqrt{x} = ۱ \longrightarrow x = ۱$ ✓

اختلاف → ۲ − ۱ = $\boxed{۱}$