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$$f(n) = \begin{cases} \cot \frac{\pi n}{r} & ; n \leq 1 \\ \sqrt{n^2+1} & ; n > 1 \end{cases}$$

$$f \circ f\left(\frac{r}{r}\right) \Rightarrow \cot \frac{r \cdot \pi}{r} = \cot \frac{\pi}{r} = \sqrt{r}$$

$$f(\sqrt{r}) = \sqrt{r+1} = (r) \checkmark$$

۲) -۱

$$f\left(\frac{1}{n}\right) = \sqrt{\frac{r \cdot n - 1}{n^2}}, \quad g(n) = r \cos n \quad f \circ g\left(\frac{\pi}{r}\right) = ?$$

$$g\left(\frac{\pi}{r}\right) = r \times \frac{1}{r} = 1 \quad f\left(\frac{1}{r}\right) \Rightarrow \sqrt{\frac{r-1}{r^2}} = \sqrt{\frac{r-1}{r}} = \left(\frac{\sqrt{r-1}}{r}\right) \checkmark$$

۲) الف -۲

$$f(n) = [n] \quad g(n) = \frac{n}{1-n} \quad f \circ g(\sqrt{r}) = ?$$

$$g(\sqrt{r}) = \frac{\sqrt{r}}{1-\sqrt{r}} \times \frac{1+\sqrt{r}}{1+\sqrt{r}} = \frac{\sqrt{r}+r}{-1} = -\sqrt{r}-r$$

$$f(-\sqrt{r}-r) = [-\sqrt{r}-r] = [-r] = (-r) \checkmark$$

$$f(n) = \sin n \quad g(n) = n \sqrt{1-n^2} \quad g \circ f\left(\frac{\pi}{r}\right) = ?$$

$$f\left(\frac{\pi}{r}\right) = \sin\left(\frac{\pi}{r}\right) = \frac{\sqrt{r}}{r} \rightarrow g\left(\frac{\sqrt{r}}{r}\right) = \frac{\sqrt{r}}{r} \sqrt{1-\frac{r}{r^2}} = \frac{\sqrt{r}}{r} \times \sqrt{\frac{r-1}{r}} = \frac{\sqrt{r}}{r} \times \frac{\sqrt{r-1}}{\sqrt{r}} = \frac{\sqrt{r-1}}{r} \checkmark$$

۲) -۳

$$f(n) = \{(1, \omega), (4, \omega), (1, \omega), (1, \omega)\} \quad g(n) = \{(1, 1), (2, 1), (1, 1), (1, 1)\}$$

$$f \circ g(n) = \{(1, \omega), (2, \omega), (1, \omega), (1, \omega)\} \checkmark$$

$$g \circ f(n) = \emptyset \checkmark$$

$$f \circ f(n) = \emptyset \checkmark$$

$$g \circ g(n) = \{(1, 1), (2, 1), (1, 1), (1, 1)\} \checkmark$$

۲) -۴

$$f = \{(1, 1), (2, 2), (1, \omega), (1, \omega)\} \quad g = \{(1, 2), (1, 1), (1, 1), (1, 1)\}$$

$$(1, 2) \in f \circ g \quad (1, 1) \in g \circ f \quad (1, 1) \in f \rightarrow (1, \omega)$$

$$g\left(\frac{f(1)}{1}\right) = 1 \Rightarrow b = 1 \checkmark$$

$$f\left(\frac{g(1)}{1}\right) \Rightarrow a = 1 \checkmark$$

۲) -۵

$$f, g \rightarrow \text{تابع خطی} \quad f(f(n)) = 5n + 12 \quad g(rn + r) = rn - r \quad g \circ f(-1) = ?$$

۲) -۶

$$rn + r = 2$$

$$n = \frac{2-r}{r}$$

$$g(z) = \frac{rz - 12}{r} \Rightarrow g\left(\frac{2-r}{r}\right) = \frac{r}{r}n - \frac{12}{r}$$

$$f(n) = an + b$$

$$f(an + b) = a(an + b) + b = a^2n + ab + b = a^2n + ab + b = a^2n + (ab + b) = a^2n + (a+1)b$$

$$a^2 = 5 \rightarrow a = \pm \sqrt{5}$$

$$ab + b = 12 \begin{cases} a = \sqrt{5} \rightarrow b = 1 \rightarrow f(n) = \sqrt{5}n + 1 \rightarrow f(-1) = -1 \\ a = -\sqrt{5} \rightarrow b = -1 \rightarrow f(n) = -\sqrt{5}n - 1 \rightarrow f(-1) = 1 \end{cases}$$

$$= 5n + 12 \leftarrow$$

$$g(f(-1)) = g(-1) = \frac{-r}{r} - \frac{12}{r} = -1 - \frac{12}{r} = (-1) \checkmark$$

۲)

$$f(n) = \sqrt{n+1} \quad g(n) = \frac{1}{n^2 - rn}$$

$$g \circ f = \text{?}$$

-۷

$$Df = \begin{cases} \sqrt{r}n & n \geq 0 \\ 0 & n < 0 \end{cases} \rightarrow Df = \mathbb{R}$$

$$Dg \Rightarrow n(n-r) \neq 0 \Rightarrow n \neq 0, r$$

$$f(n) \neq 0, r \begin{cases} f(n) \neq 0 \rightarrow (0, r) \\ f(n) \neq r \rightarrow \mathbb{R} - \{n\} \end{cases}$$

$$\Rightarrow D(g \circ f) = (0, r) - \{n\} \checkmark$$

$$f(n) = \sqrt{1-n^2}, \quad g(n) = \sqrt{n}$$

$$(f+g) \circ f = ?$$

$$Df \rightarrow 1-n^2 \geq 0 \rightarrow -1 \leq n \leq 1 \rightarrow Df = [-1, 1] \quad Df+g = [0, 1]$$

$$Dg \rightarrow n \geq 0 \rightarrow Dg = [0, +\infty)$$

$$0 \leq \sqrt{1-n^2} \leq 1$$

$$\checkmark D(f+g) \circ f = [-1, 1]$$

$$\text{الف) } f\left(\frac{r(n+1)}{n-r}\right) = rn + a$$

$$\frac{r(n+1)}{n-r} = z \rightarrow zn - rz = r(n+1)$$

$$f(z) = f\left(\frac{r(z+1)}{z-r}\right) + a = \frac{1(z+1)}{z-r} + a \rightarrow f(z) = \frac{r(z+1)}{z-r}$$

$$\frac{r(z-1)}{z-r}$$

$$f(n) = \frac{r(n-1)}{n-r} \checkmark$$

$$\text{ب) } f\left(n + \frac{1}{n}\right) = n^r + \frac{1}{n}$$

$$n + \frac{1}{n} = z \quad \left(n + \frac{1}{n}\right)^r = n^r + rn + \frac{r}{n} + \frac{1}{n^r}$$

$$\left(n^r + \frac{1}{n^r}\right) = \left(n + \frac{1}{n}\right)^r - r\left(n + \frac{1}{n}\right) \quad \frac{n + \frac{1}{n} = z}{f(z) = z^r - rz} \rightarrow f(n) = n^r - rn \checkmark$$

$$f(n) = 1 \rightarrow \sqrt{n} = 1$$

$$f(n) = r\sqrt{r} \rightarrow \sqrt{n} = r\sqrt{r}$$

$$r = 1 = 1 \checkmark$$

القيمة فقط متطابقة