

$$f \circ f(n) = f(f(n)) \Rightarrow f \circ f\left(\frac{1}{\sqrt{3}}\right) = f\left(f\left(\frac{1}{\sqrt{3}}\right)\right) \Rightarrow$$

$$f\left(\frac{1}{\sqrt{3}}\right) = \cot \frac{\pi}{4} = \cot \frac{\pi}{6} = \sqrt{3}$$

$$\Rightarrow f(\sqrt{3}) = \sqrt{3+1} = 2$$

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$$\sqrt{\frac{2n-1}{2n}} = f\left(\frac{1}{2n}\right) = \sqrt{\frac{2}{2n} - \frac{1}{2n}} \Rightarrow f(n) = \sqrt{2n - n^2}$$

$$f \circ g\left(\frac{n}{2}\right) = \sqrt{\frac{n}{2}} = \left(\frac{\sqrt{n}}{2}\right)^2$$

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$$g(\sqrt{2}) = \frac{\sqrt{2}}{-\sqrt{2}} = \frac{\sqrt{2}+2}{-1} = -\sqrt{2}-2 \quad f(-2-\sqrt{2}) = [-2-\sqrt{2}]$$

$$-2 + [-\sqrt{2}] = -2-\sqrt{2} = -\sqrt{2}$$

$$\sqrt{1} < \sqrt{2} < \sqrt{3}$$

$$1 < \sqrt{2} < \sqrt{3} \Rightarrow -\sqrt{2} < -3$$

$$\frac{\sqrt{2}}{2} = f\left(\frac{n}{2}\right) \quad \frac{\sqrt{2}}{2} \sqrt{1-\frac{1}{2}} = \left(\frac{1}{2}\right)$$

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$$g\left(f\left(\frac{n}{2}\right)\right) = g\left(\frac{\sqrt{2}}{2}\right)$$

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$$۱) f \circ g(n) = \{(4, 5), (2, 5), (6, 12), (8, 12)\}$$

$$۲) g \circ f(n) = \{ \}$$

$$۳) f \circ f(n) = \{ \}$$

$$۴) g \circ g(n) = \{(4, 1), (2, 6), (6, 1), (8, 1)\}$$

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$$f \circ g(a) = 2 \Rightarrow f(g(a)) = 2 \Rightarrow g(a) = 3$$

$$\Rightarrow \boxed{a=4} \quad (4, 1) \quad g \circ f(4) = 1$$

$$\Rightarrow g(f(a)) = g(b) \Rightarrow f(a) = b \Rightarrow \boxed{b=a}$$

$$(a, b) = (4, 1)$$

$$f(x) = a \cdot (an + b) + b = f_{n+1} \Rightarrow an + ab + b = f_{n+1}$$

$$\Rightarrow a = \frac{f_{n+1} - b}{n + b} \Rightarrow \frac{f_{n+1} - b}{n + b} = \frac{f_n - b}{n + b}$$

$$\Rightarrow \frac{f_{n+1} - b}{n + b} = \frac{f_n - b}{n + b} \Rightarrow f_{n+1} = f_n$$

$$\Rightarrow f(-1) = \frac{0}{-1} = 0$$

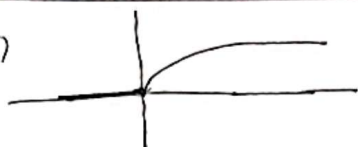
$$f_n + 1 = 0 \Rightarrow n = -1$$

$$f_n + 1 = -1 \Rightarrow n = -2$$

$$\Rightarrow g \circ f(-1) = \frac{-1}{-1} = 1$$

$$D_f = \mathbb{R}$$

$$R_f = [0, +\infty)$$



$$\Rightarrow \sqrt{n+1} \neq 0, \forall$$

$$g(n) = \frac{1}{n^2 - f_n} \Rightarrow + \frac{1}{n^2} - \frac{1}{n^2} +$$

$$\Rightarrow n > 0$$

$$\Rightarrow n \neq 1$$

$$\Rightarrow D_{g \circ f} = (0, +\infty) - \{1\}$$

$$f(x) = \sqrt{1-x^2} \Rightarrow \frac{-1}{2\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}} \Rightarrow D_f = [-1, 1]$$

$$\Rightarrow R_f = [0, 1] \quad g(n) = \sqrt{n} \Rightarrow n \in [0, +\infty)$$

$$(f+g) \circ f \Rightarrow (f+g)([0, 1]) \Rightarrow D_{(f+g) \circ f} = [0, 1]$$

$$D_f \cap D_g \cap R_f = [0, 1]$$

$$\omega) \quad \frac{f_{n+1}}{f_n} = p \Rightarrow \frac{f_{n+1}}{f_n} = p \Rightarrow f(n) = f\left(\frac{1+p}{n-p}\right) + d$$

$$\therefore) \quad f\left(n + \frac{1}{n}\right) = n + \frac{1}{n} \Rightarrow \left(n + \frac{1}{n}\right)^p$$

$$= n^p + \frac{1}{n^p} + p\left(\frac{1}{n} + n\right) \Rightarrow f(n) = n^p - \frac{1}{n^p}$$

$$g \circ f(n) = g(f(n))$$

$$g(n) = 0 \Rightarrow n = \frac{1}{p}$$

$$f(n) = \sqrt{p} > 1$$

$$n\sqrt{n} = \sqrt{p} \Rightarrow n^{\frac{3}{2}} = \sqrt{p} \Rightarrow n = \sqrt{p}$$

$$p-1 = 1$$

$$n\sqrt{n} = 1 \Rightarrow n^{\frac{3}{2}} = 1 \Rightarrow n = 1$$