نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۹ ..... کلاس ..... B .....

۱

$f\circ f(x)=f(f(x)) \Rightarrow f\circ f\left(\frac{\pi}{20}\right)=f\left(f\left(\frac{\pi}{20}\right)\right) \Rightarrow$

$f\left(\frac{\pi}{20}\right) \to \cot\frac{2\pi}{6}=\cot\frac{\pi}{3}=\sqrt{3}$

$\Rightarrow f(\sqrt{3})=\sqrt{3+1}=2$

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$\sqrt{\frac{2x-1}{x^2}}=f\left(\frac{1}{x}\right)=\sqrt{\frac{2}{x}-\frac{1}{x^2}} \Rightarrow f(x)=\sqrt{2x-x^2}$

$f\circ g\left(\frac{\pi}{3}\right)=\sqrt{\frac{3}{2}}=\frac{\sqrt{3}}{2}$ (الف)

$g(\sqrt{2})=\frac{\sqrt{2}}{-\sqrt{2}}=\frac{\sqrt{2}+2}{-1}=-\sqrt{2}-2$ (ب)

$f(-2-\sqrt{2})=[-2-\sqrt{2}]$

$\sqrt{1}<\sqrt{2}<\sqrt{4}$

$1<\sqrt{2}<2 \Rightarrow -3<-\sqrt{2}<-2$

$-2+[-\sqrt{2}]=-2-2=-4$

۳

$\frac{\sqrt{2}}{2}=f\left(\frac{\pi}{4}\right)$ $\frac{\sqrt{2}}{2}\sqrt{1-\frac{1}{2}}=\frac{1}{2}$

$g\left(f\left(\frac{\pi}{4}\right)\right)=g\left(\frac{\sqrt{2}}{2}\right)$

۴

الف) $f\circ g(x)=\{(4,3),(2,5),(6,12),(8,12)\}$

ب) $g\circ f(x)=\{\}$

ج) $f\circ f(x)=\{\}$

د) $g\circ g(x)=\{(4,8),(2,6),(6,10)\}$

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$f\circ g(1)=2 \Rightarrow f(g(1))=2 \Rightarrow g(1)=3$

$\Rightarrow \boxed{a=4}$ $g\circ f(4)=1$

$\Rightarrow g(f(4))=g(b) \Rightarrow f(1)=b \Rightarrow \boxed{b=5}$

$(a,b)=(4,5)$

$f(f(x)) = a(ax+b)+b = 4x+3 \Rightarrow a^2x+ab+b = 4x+3$

$\Rightarrow a = +2 \to b = 1$, $a = -2 \to b = -3$ $\Rightarrow f(x) = 2x+1$ or $-2x-3$

$\Rightarrow f(-1) = 0$ or -5 , $2x+3=0 \to x=-\frac{3}{2}$, $2x+3=-5 \Rightarrow x=-4$

$\Rightarrow g \circ f(-1) = \frac{-13}{2}$ or -17

$D_f = \mathbb{R}$

$R_f = [0, +\infty)$

$\Rightarrow \sqrt{x+|x|} \neq 0, 2$

$g(x) = \frac{1}{x^2-4x}$ $\Rightarrow$ + 0 − 4 +

$\Rightarrow x > 0$

$\Rightarrow x \neq 2$

$\Rightarrow D_{g \circ f} = (0, +\infty) - \{2\}$

$f(x) = \sqrt{1-x^2}$ $\Rightarrow$ − 0 + 0 −

$D_f = [-1, 1]$

$\Rightarrow R_f = [0, 1]$

$g(x) = \sqrt{x} \Rightarrow x \in [0, +\infty)$

$(f+g) \circ f \Rightarrow (f+g)([0,1]) \to D_{(f+g)\circ f}$

$= [0,1]$ $D_f \cap D_g \cap R_f = [0,1]$

ب) $\frac{2x+1}{x-2} = p \Rightarrow \frac{2p+1}{p-2} = x \Rightarrow f(x) = f\left(\frac{1+2x}{x-2}\right) + 5$

$=$ [illegible]

ج) $f\left(x+\frac{1}{x}\right) = x^3 + \frac{1}{x^3}$ [illegible] $\Rightarrow \left(x+\frac{1}{x}\right)^3$

$= x^3 + \frac{1}{x^3} + 3\left(\frac{1}{x}+x\right) \Rightarrow \boxed{f(x) = x^3 - 3x}$

$g \circ f(x) = g(f(x))$

$g(x) = 0 \Rightarrow x = \sqrt{2}, 1$

$f(x) = 2\sqrt{2}, 1$

$x\sqrt{x} = 2\sqrt{2} \Rightarrow x^{\frac{3}{2}} = 2^{\frac{3}{2}} \Rightarrow \boxed{x=2}$ $\qquad 2-1 = \boxed{1}$

$x\sqrt{x} = 1 \Rightarrow x^{\frac{3}{2}} = 1^{\frac{3}{2}} \Rightarrow \boxed{x=1}$