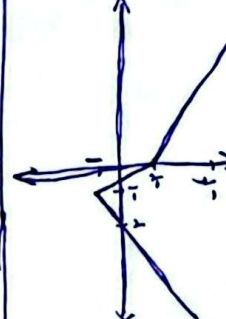


سوال ۱) $f(x) = \sqrt{1-x^2}$, $g(x) = \{(-1,1), (0,0), (1,1)\}$
 $fg - xf = 9 \rightarrow f_{(1)} = (-1,0), f_{(0)} = (0,1), f_{(1)} = (1,-3) = f_{(1)}$ و غیره
 $\Rightarrow 3f = (-1,0), (0,3), (1,0)$ و $2g = (-1,2), (0,8), (1,4)$
 $2g - 3f = (-1,2), (0,5), (1,4) \Rightarrow 2 + 5 + 4 = 11$

سوال ۲) $f(x) = 2x - 1 \xrightarrow{\text{دایره}} [3, +\infty) \rightarrow f_{(1)} = 5 : \text{پس}$
 $g(x) = \frac{1}{3}x + 3 \xrightarrow{\text{دایره}} (-\infty, 3] \rightarrow g_{(3)} = 6 : \text{پس}$
 $\Rightarrow \text{اجتماع}$

سوال ۳) $y = \frac{-a^2}{y} + a + 3 \rightarrow \text{دایره}$ و $\frac{1}{y} = \frac{1}{a} \Rightarrow a = y$
 $a = 3 \rightarrow -f, 5 + 3 + 3 = 15$
 $\Rightarrow R_f \geq 1, 5 \rightarrow D_f = (-1, 3) \rightarrow \sqrt{b-a} = \sqrt{3-(-1)} = \sqrt{4} = 2$

سوال ۴) $y = |a-1| + |a-3| + |2a-4| \rightarrow a < 1 \rightarrow 1-2a \rightarrow y = 2$
 $a > 3 \rightarrow 1 \rightarrow y = 2$
 $1 < a < 3 \rightarrow 4-2a \rightarrow y = 2$
 $2 < a < 3 \rightarrow 2a-2 \rightarrow y = 2$
 $a \geq 3 \rightarrow 2a-4 \rightarrow y = 2$

سوال ۵) $y = |a-1| - 2|a+1|$


سوال ۶) $y = \frac{a^2 + 5a + m}{a+1}$
 $y_a + y = a^2 + 5a + m \rightarrow a^2 + (5-y)a + m = 0 \rightarrow \Delta \geq 0 \rightarrow (5-y)^2 - 4m \geq 0$
 $\rightarrow y^2 - 10y + 25 - 4m + 4y \geq 0 \rightarrow y^2 - 6y + 25 - 4m \geq 0 \rightarrow R_f = 8 \Rightarrow \Delta \leq 0$
 $\rightarrow 3y - 10 + 14m \leq 0 \rightarrow m \leq \frac{10-3y}{4} \rightarrow m = f \rightarrow y = \frac{a^2 + 5a + f}{a+1} \Rightarrow a + f = R_f \Rightarrow a = R_f - f$

سوال ۷) $f(a) = \begin{cases} a+2 & ; a \geq 3 \\ a^2 - 4a + 2 & ; 0 \leq a < 3 \\ |a| + 2 & ; a < 0 \end{cases} \rightarrow f_{(1)} = 2, f_{(3)} = 5 \rightarrow [1, 5]$
 $\Rightarrow [1, +\infty)$

$f(x) = \begin{cases} x^2 + (a+r)x > 0 \rightarrow f(-r) = -1, f(0) = r \rightarrow [-1, +\infty) \\ [ra] - r < 0 \rightarrow [-1, 0] \end{cases}$
 $\rightarrow x \in [-1, +\infty)$

مثال 18

$f(x) = a + 1 - \sqrt{ra + r} \rightarrow ra > -r \rightarrow a > -\frac{r}{r} \xrightarrow{D_F} [-\frac{r}{r}, +\infty) \rightarrow \sqrt{bz - \frac{r}{r}}$
 $\rightarrow a + 1 - \sqrt{ra + r} \rightarrow \sqrt{r(-\frac{r}{r}) + r} \rightarrow a + 1 \rightarrow \text{كثير و كثير} \rightarrow (-\infty, a + 1]$
 $\rightarrow a + 1 \leq d \rightarrow [a, d]$
 $\Rightarrow ab \leq -\frac{r}{r} \times f \leq -\frac{r}{r}$

مثال 19

$f(x) = \sqrt{a+1} + f\sqrt{1-a} \rightarrow D_F \in [-1, 1]$
 $f(a) + f(a) = \sqrt{r+1}\sqrt{1-a} \rightarrow f(a) = \sqrt{r+1}\sqrt{1-a} - f(a)$
 $y = \frac{f(a)}{r} - \sqrt{r+1}\sqrt{1-a} + \frac{f(a)}{r}$
 $D \in [-1, 1] \rightarrow \frac{f(a)}{r} - \sqrt{r+1}\sqrt{1-a} \leq \sqrt{a+1} + r\sqrt{1-a} - \sqrt{r+1}\sqrt{1-a}$
 $\rightarrow a + rb - \frac{ra + r + ab}{|a+b|} \leq b \leq \sqrt{1-a}$
 $\sqrt{a+1} \leq a$
 $\sqrt{1-a} \leq b$
 $a + b > 0$
 $\sqrt{0, r} \rightarrow R[0, r]$

مثال 20