Subject.

Date.

تکلیف ۵ - امیرپویا رزاق - یازدهم بیان B

① $f(x) = \sqrt{1-x^2} \Rightarrow 1-x^2 \geq 0 \Rightarrow -1 \leq x \leq 1 \Rightarrow x = -1, 0, 1$

$f(-1) = \sqrt{1-(-1)^2} = \sqrt{0} = 0 \quad g(-1) = 1 \,,\; g(0) = 4 \,,\; g(1) = 2$

$f(0) = 1 \quad f(1) = 0$

$\Rightarrow 2g - 3f \xrightarrow{x=-1} 2(1) - 3(0) = 2 \,,\; x=0 \rightarrow 2(4) - 3(1) = 5$

$\xrightarrow{x=1} 2(2) - 3(0) = 4 \Rightarrow \boxed{R = \{2, 4, 5\}}$

② $f(x) = 2x - 1 \,,\; D_f = [3, +\infty) \,,\; g(x) = \frac{1}{3}x + 3 \,,\; D_g = (-\infty, 3]$

$f(3) = 5 \Rightarrow R_f = [5, +\infty) \Big\} \; g(3) = 4 \,,\; R_g = (-\infty, 4]$

↳ کمترین مقدار دامنه

$\boxed{R_f \cup R_g = (-\infty, 4] \cup [5, +\infty)}$

③ $y = \frac{-x^2}{2} + x + 3 \Rightarrow$

$\frac{-x^2}{2} + x + 3 > \frac{3}{2} \Rightarrow \frac{-x^2}{2} + x + \frac{3}{2} > 0 \Rightarrow x^2 - 2x - 3 < 0$

-1		3
+	-	+

$\Rightarrow -1 < x < 3 \Rightarrow x \in (-1, 3) \Rightarrow \begin{cases} b = 3 \\ a = -1 \end{cases} \Rightarrow \sqrt{b-a} = \sqrt{3-(-1)} = \sqrt{4} = 2$

$\Rightarrow \boxed{\sqrt{b-a}_{max} = 2}$

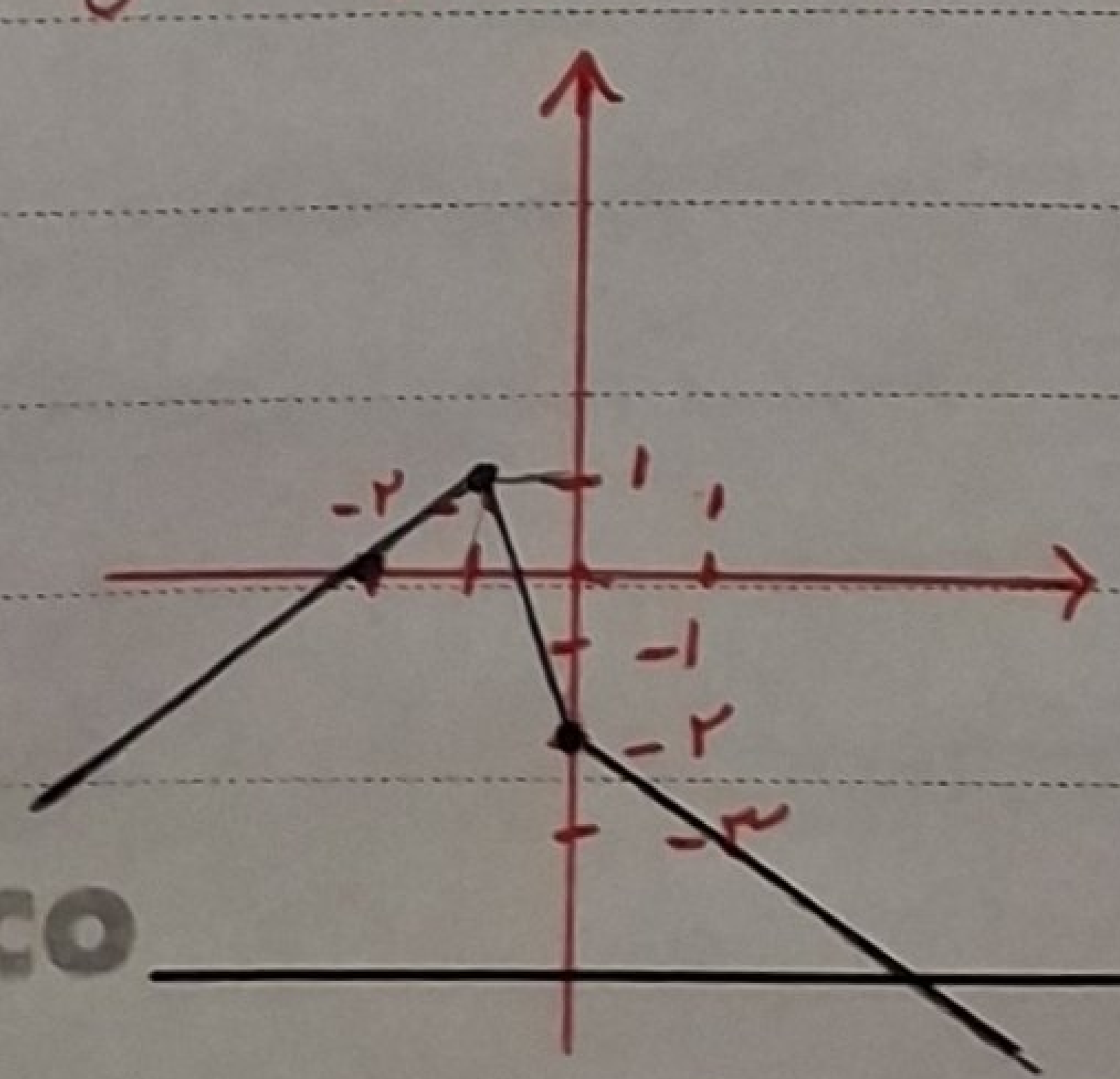
④ $y = |x-1| + |x-3| + |2x-4| \Rightarrow y = |x-1| + |x-3| + 2|x-2|$

$x \geq 3 \rightarrow y = 4x - 8 \xrightarrow{x=3} y = 4 \,,\quad 2 < x \leq 3 \rightarrow y = 2x - 2 \xrightarrow{x=2} y = 2$

$1 \leq x \leq 2 \rightarrow y = 6 - 2x \xrightarrow{x=1} y = 4 \,,\; x \leq 1 \rightarrow y = 8 - 4x \xrightarrow{x=1} y = 4$

$\Rightarrow \boxed{y_{min} = 2}$

⑤ $y = |x| - 2|x+1| \Rightarrow y = \begin{cases} x - 2x - 2 & x > 0 \\ -x - 2x - 2 & -1 \leq x \leq 0 \\ -x + 2x + 2 & x < -1 \end{cases} \Rightarrow \begin{cases} -x - 2 & x \geq 0 \\ -3x - 2 & -1 \leq x \leq 0 \\ x + 2 & x \leq -1 \end{cases}$



$\boxed{R_f = (-\infty, 1]}$

Subject.

Date.

⑥

$y = \frac{x^2 + 5x + m}{x+1}$ $(m \in \mathbb{N}) \Rightarrow xy + y = x^2 + 5x + m \Rightarrow x^2 + (5-y)x + (m-y) = 0$

$\Delta = (5-y)^2 - 4(m-y) \geq 0 \Rightarrow y^2 - 10y + 25 - 4m + 4y \geq 0$

$\Rightarrow y^2 - 6y + (25 - 4m) \geq 0 \Rightarrow \Delta_{min} = (3)^2 - 6(3) + (25 - 4m) \geq 0$

$\Rightarrow -9 + 25 - 4m \geq 0 \Rightarrow 16 - 4m \geq 0 \Rightarrow m \leq 4 \Rightarrow m = 1, 2, 3, 4 \Rightarrow$

$R = \mathbb{R}$

⑦ $f(x) = \begin{cases} x+2 ; & x \geq 2 \quad \text{شرط ۱} \\ x^2 - 2x + 2 ; & 0 \leq x < 2 \quad \text{شرط ۲} \\ |x| + 2 ; & x < 0 \quad \text{شرط ۳} \end{cases}$

① $x \geq 2 \rightarrow x+2 \Rightarrow [4, +\infty)$ ، ② $x^2 - 2x + 2 = (x-1)^2 + 1 \Rightarrow [1, 4)$

③ $x = 0 \rightarrow (2, +\infty) \Rightarrow R_f = ① \cup ② \cup ③ = [1, +\infty)$

⑧ $f(x) = \begin{cases} x^2 + 4x + 2 ; & x < 0 \\ [2x] - 2x ; & x \geq 0 \end{cases}$ $x^2 + 4x + 2 \Rightarrow R = [y_{min}, +\infty) = [-1, +\infty)$

$[2x] - 2x \rightarrow [t] - t \Rightarrow -1 < [t] - t \leq 0 \Rightarrow R_f = ① \cup ② =$

$\Rightarrow R_f = [-1, +\infty)$

⑨ $y = a + 1 - \sqrt{-1, +\infty}$ $D_f = [b, +\infty)$ $R_f = (-\infty, 4]$

$D_f = 2x + 2 \geq 0 \Rightarrow x \geq \frac{-2}{2} \Rightarrow b = -\frac{2}{2}$ $y = a + 1 - \sqrt{0} = a + 1$

$\Rightarrow a + 1 = 4 \Rightarrow a = 3$ ، $a = 3$ ، $b = -\frac{2}{2} \Rightarrow ab = (3)(-\frac{2}{2}) = -3$

$\Rightarrow ab = -3$

PAPCO

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Date.

(10) $f(a)+g(a)=\sqrt[3]{2+\sqrt[3]{1-x^3}} \Rightarrow \sqrt[3]{2+\sqrt[3]{1-x}\cdot\sqrt{x+1}} \Rightarrow \sqrt[3]{(\sqrt{x+1})^3+\sqrt{1-x}}$

$f(x)+g(x)=\sqrt[3]{x+1}+\sqrt[3]{1-x}$; $f(x)=2\sqrt{x+1}+4\sqrt{1-x}$

$g(x)=(\sqrt[3]{x+1}+\sqrt[3]{1-x})-(2\sqrt{x+1}+4\sqrt{1-x})=\sqrt{x+1}-\sqrt{1-x}$

$y=\frac{f(x)}{4}-\frac{g(x)}{2} \Rightarrow \frac{2\sqrt{x+1}+\cancel{4}^{2}\sqrt{1-x}}{\frac{4}{2}}-\frac{\sqrt{x+1}-\sqrt{1-x}}{2}$

$\Rightarrow y=\frac{\sqrt{x+1}+2\sqrt{1-x}-\sqrt{x+1}+\sqrt{1-x}}{2}=\frac{2\sqrt{1-x}}{2}=\sqrt{1-x}$

$D_f=D_g=[-1,1]$, $R=[0,\sqrt{2}]$

$\Rightarrow y=\sqrt{1-x}$, $D=[-1,1]$, $R=[0,\sqrt{2}]$

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