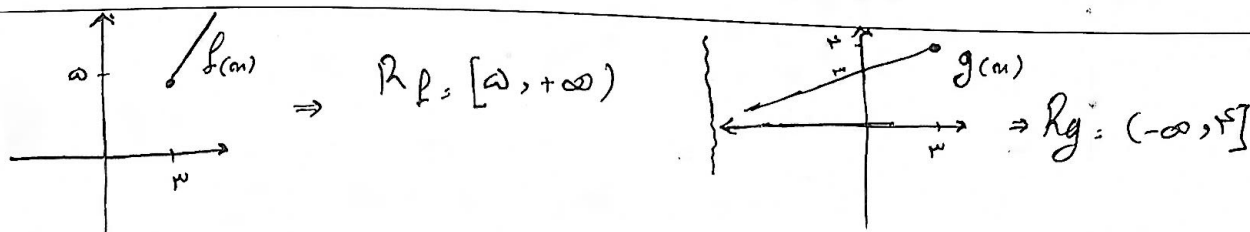


$$|a| \leq 1 \Rightarrow -1 \leq a \leq 1 \Rightarrow D_f = [-1, 1] \quad D_f \cap D_g = \{-1, 0, 1\}$$

$$\Rightarrow f \cdot g = \{(-1, 2), (0, 5), (1, 4)\} \quad R_{f \cdot g} = \{2, 5, 4\}$$

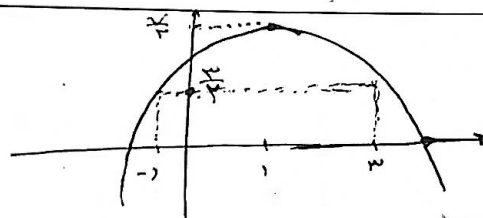
$$\Rightarrow 2 + 5 + 4 = 11$$



$$\Rightarrow R_f \cup R_g = (-\infty, 4] \cup [5, +\infty) \subset \mathbb{R} - (4, 5)$$

$$y = -\frac{1}{4}x^2 + x + 3$$

$$\max_{x \in \mathbb{R}} \begin{cases} -\frac{b}{2a} = \frac{-1}{-1/2} = 2 \\ -\frac{1}{4} + 4 + 3 = \frac{25}{4} \end{cases}$$

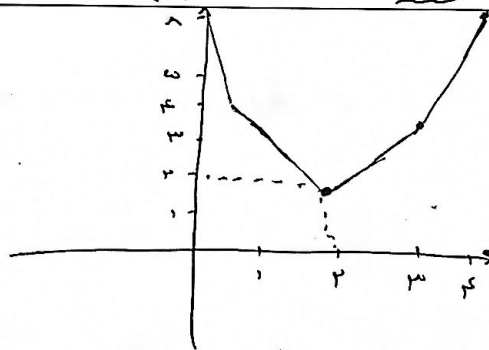


$$-\frac{1}{4}x^2 + x + 3 = \frac{25}{4} \Rightarrow -\frac{1}{4}x^2 + x + \frac{3}{4} = 0 \Rightarrow x = -1, 3 \Rightarrow (a, b) = (-1, 3)$$

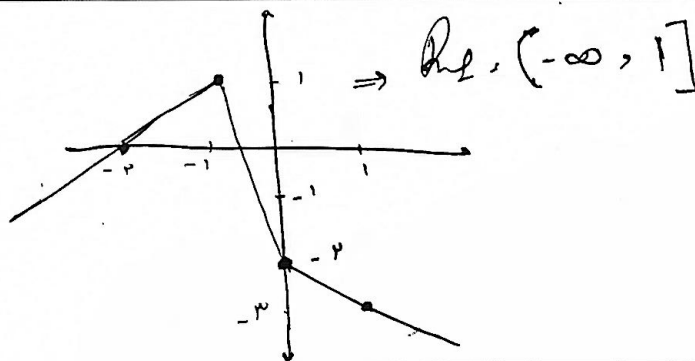
$$\Rightarrow \sqrt{b-a} = \sqrt{3-(-1)} = \sqrt{4} = 2$$

1	2	3
$-f(a) + 1$	$-f(a) + 9$	$f(a) - 2$

$$\Rightarrow \min = 2$$



-1	0
$a + 2$	$-3a - 2$



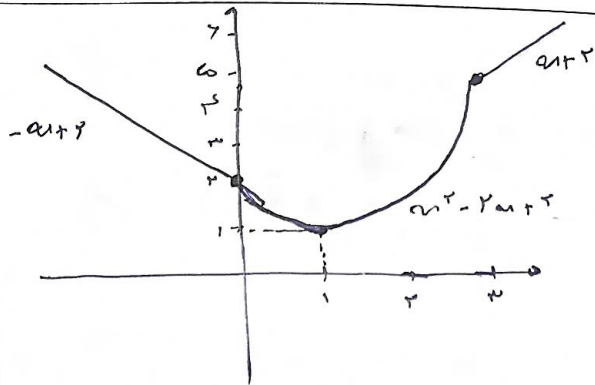
$$a_1^2 + a_1 + m = y a_1 + y \Rightarrow a_1^2 + (a_1 - y) a_1 + (m - y) = 0 \Rightarrow \Delta \geq 0$$

$$\Rightarrow (a_1 - y)^2 - 4(m - y) \geq 0 \Rightarrow a_1^2 + y^2 - 10y - 4m + 4y \geq 0 \Rightarrow y^2 - 6y + (2a_1 - 4m) \geq 0$$

چون عبارت درجه دوم بزرگتر مساوی صفر است پس دلتای آن که چک می‌کنیم باید باشد.

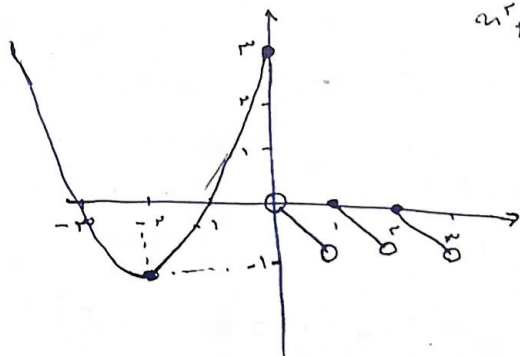
$$\Rightarrow \Delta \leq 0 \Rightarrow (-6)^2 - 4(2a_1 - 4m) \leq 0 \Rightarrow 36 - 10a_1 + 16m \leq 0 \Rightarrow 16m \leq 10a_1 - 36 \Rightarrow m \leq \frac{5a_1 - 9}{8}$$

$$\Rightarrow m = \{1, 2, 3, 4\} \Rightarrow \text{به ازای ۵ مقدار طبیعی}$$



$$\min \left| \begin{array}{l} \frac{-b}{2a} = \frac{y}{x} = 1 \\ 1 - 2 + 2 = 1 \end{array} \right|$$

$$\Rightarrow R_f: [1, +\infty)$$



$$a_1^2 + 4a_1 + 3 = (a_1 + 1)(a_1 + 3)$$

$$\min \left| \begin{array}{l} \frac{-b}{2a} = \frac{-4}{2} = -2 \\ (-2)^2 + 4(-2) + 3 = -1 \end{array} \right|$$

$$\Rightarrow R_f: [-1, +\infty)$$

$$2a_1 \geq 3 \Rightarrow a_1 \geq \frac{3}{2} \Rightarrow R_f = [\frac{3}{2}, +\infty) = [b, +\infty) \Rightarrow b = \frac{3}{2}$$

$$\max = a + 1 - \sqrt{2(\frac{3}{2}) + 3} = a + 1 = 4 \Rightarrow a = 3$$

$$\Rightarrow ab = 3 \times \frac{3}{2} = \frac{9}{2}$$

$$2x \geq -1, 1 \geq a \Rightarrow R_f, R_g: [-1, 1]$$

$$f + 2\sqrt{1-a}x, (\sqrt{1-a} + \sqrt{1+a})^2$$

$$\Rightarrow 2\sqrt{1-a} + 2\sqrt{1+a}, 2(\sqrt{1-a} + \sqrt{1+a}) = f(m) + g(m) \Rightarrow g(m), 2\sqrt{1-a} + 2\sqrt{1+a} - 2\sqrt{1+a} - 2\sqrt{1-a}$$

$$\Rightarrow g(m), \sqrt{1+a} - \sqrt{1-a} \Rightarrow g: \frac{f(m) - 2g(m)}{4} \Rightarrow g: \frac{9\sqrt{1-a}}{4} = \sqrt{1-a}$$

$$D: [-1, 1] \rightarrow R: [0, \sqrt{2}]$$