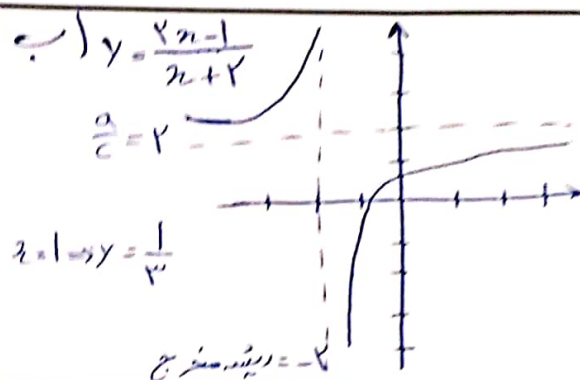
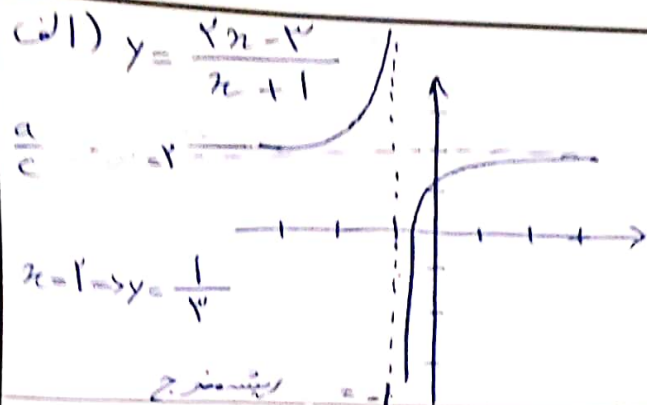




ا) $y = \sin x + \frac{1}{\sin x} \Rightarrow R_f = (-\infty, -1] \cup [1, +\infty)$

ب) $y = \frac{x^4+1}{x^2} \Rightarrow y = x^2 + \frac{1}{x^2} \Rightarrow R_f = (-\infty, -1] \cup [1, +\infty)$

ج) $y = \frac{\sqrt[3]{2x+1}}{\sqrt[3]{2}} \Rightarrow y = \sqrt[3]{x} + \frac{1}{\sqrt[3]{2}} \Rightarrow R_f = (-\infty, -1] \cup [1, +\infty)$

د) $y = \sqrt{x} + \frac{1}{\sqrt{x}} \Rightarrow R_f = [1, +\infty)$

ا) $y = x^2 + \frac{1}{x^2+1} \Rightarrow y = x^2+1 \cdot \frac{1}{x^2+1} - 1 \Rightarrow R_f = [\frac{1}{2}, +\infty)$

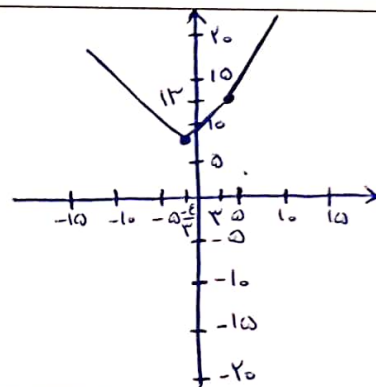
ب) $x = \frac{x^2+1}{\sqrt{x^2+1}} \Rightarrow y = \frac{x^2+1}{\sqrt{x^2+1}} \Rightarrow y = \sqrt{x^2+1} + \frac{1}{\sqrt{x^2+1}}$

$\Rightarrow R_f = [\frac{1}{2}, +\infty)$ $\Rightarrow \min y = \sqrt{\varepsilon} + \frac{1}{\sqrt{\varepsilon}} = 1 + \frac{1}{2} \Rightarrow R_f = [\frac{3}{2}, +\infty)$

$a = \sqrt{x^2+1} = 1$ عند $x=0$

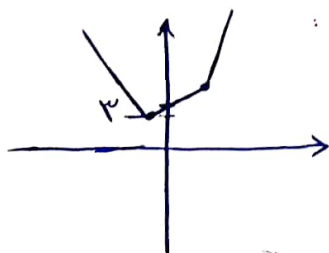
$y = |x-1| + |x+1|$

$\begin{cases} x-1+1+x+1 & x > 1 \\ 1+x+1-x-1 & 1 \geq x \geq -1 \\ -x+1-1-x-1 & x < -1 \end{cases} \Rightarrow \begin{cases} 2x+1 & x > 1 \\ 2 & 1 \geq x \geq -1 \\ -2x-1 & x < -1 \end{cases}$



ا) $y = |x-1| + |x+1|$

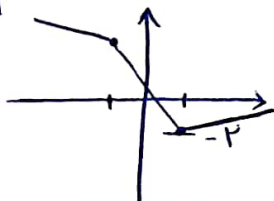
$\begin{cases} 2x & x > 1 \\ x+1 & 1 \geq x \geq -1 \\ -2x & x < -1 \end{cases}$



$R_f = [2, +\infty)$

ب) $y = |x-1| - |x+1|$

$\begin{cases} x-1 & x > 1 \\ -1-x+1 & 1 \geq x \geq -1 \\ -x+1 & x < -1 \end{cases}$



$R_f = [-1, +\infty)$

الف) $y = x^3 - 3x^2 + 3x \Rightarrow y - 1 = x^3 - 3x^2 + 3x - 1 \Rightarrow y - 1 = (x - 1)^3 \Rightarrow \sqrt[3]{y - 1} = x - 1$
 $\Rightarrow x = \sqrt[3]{y - 1} + 1 \Rightarrow R_f = \mathbb{R}$

ب) $y = \frac{1}{x^2 - 2x} \Rightarrow yx^2 - 2yx - 1 = 0 \Rightarrow \Delta = 4y^2 + 4y \Rightarrow x = \frac{2y \pm \sqrt{4y^2 + 4y}}{2y}$
 $\Rightarrow y \neq 0, 4y^2 + 4y \geq 0 \Rightarrow y^2 + y \geq 0 \Rightarrow \frac{1}{y} \left| \begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array} \right| \Rightarrow R_f = (-\infty, -1] \cup (0, +\infty)$

الف) $y = x^2 - 4x + 10 \xrightarrow{a > 0} \begin{cases} x_{\min} = \frac{-b}{2a} = 2 \\ y_{\min} = y_{\text{min}} = 6 \end{cases} \Rightarrow R_f = [6, +\infty)$

ب) $y = x^2 + 4x + 3 \xrightarrow{a < 0} \begin{cases} x_{\max} = \frac{-b}{2a} = -2 \\ y_{\max} = 1 \end{cases} \Rightarrow R_f = (-\infty, 1]$

ج) $y = \sqrt{x^2 - 4x - 2} \xrightarrow{a > 0} \begin{cases} x_{\min} = \frac{-b}{2a} = 2 \\ y_{\min} = -\sqrt{2} \end{cases} \Rightarrow R_f = \sqrt{[-\sqrt{2}, +\infty)} = [0, +\infty)$

د) $y = \sqrt{4 - x^2} \xrightarrow{a < 0} \begin{cases} x_{\max} = \frac{-b}{2a} = 0 \\ y_{\max} = 2 \end{cases} \Rightarrow R_f = \sqrt{(-\infty, 2]} \Rightarrow R_f = [0, 2]$

الف) $y = x^3 - 6x^2 + 9x + 1 \quad R_f = \mathbb{R}$

ب) $y = x^3 - 9x^2 + 13x + 2 \quad R_f = \mathbb{R}$

ج) $y = \sqrt{x^3 - 9x^2 + 13x + 1} \quad R_f = [0, +\infty)$

د) $y = (x^3 - 9x^2 + 13x + 1)^2 \quad R_f = (\mathbb{R})^2 = [0, +\infty)$

الف) $y = \frac{x+1}{x+2} \quad R_f = \mathbb{R} - \left\{\frac{a}{c}\right\} = \mathbb{R} - \left\{\frac{1}{2}\right\}$

ب) $y = \frac{2x+1}{x+1} \quad R_f = \mathbb{R} - \left\{\frac{a}{c}\right\} = \mathbb{R} - \left\{\frac{1}{1}\right\}$

الف) $y = \sqrt{\frac{x+1}{x+2}} \Rightarrow R_f = \sqrt{\mathbb{R} - \left\{\frac{1}{2}\right\}} = [0, +\infty) - \left\{\sqrt{\frac{1}{2}}\right\}$

ب) $y = \sqrt{\frac{x+1}{x-2}} \Rightarrow R_f = \sqrt{\mathbb{R} - \left\{-\frac{1}{2}\right\}} = [0, +\infty)$