

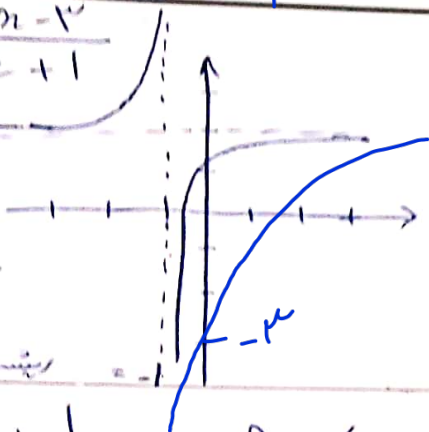
حل بر حسب نمودار با محورهای مختصات

الف) $y = \frac{x^3 - 1}{x + 1}$

$\frac{a}{c} = 1$

$x = 1 \Rightarrow y = \frac{1}{2}$

ریشه منفرجه

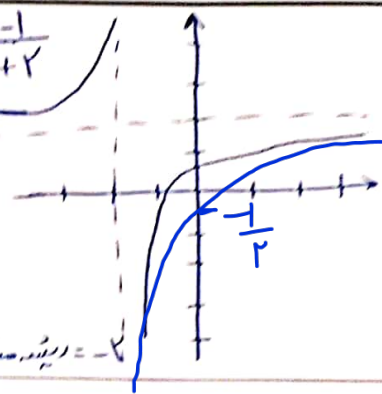


ب) $y = \frac{x^3 - 1}{x + 2}$

$\frac{a}{c} = 1$

$x = 1 \Rightarrow y = \frac{1}{3}$

ریشه منفرجه



الف

الف) $y = \sin x + \frac{1}{\sin x} \Rightarrow R_f = (-\infty, -2] \cup [2, +\infty)$ ✓

ب) $y = \frac{x^4 + 1}{x^3} \Rightarrow y = x + \frac{1}{x^3} \Rightarrow R_f = (-\infty, -2] \cup [2, +\infty)$ ✓

ج) $y = \frac{\sqrt[3]{x^3 + 1}}{\sqrt[3]{x}} \Rightarrow y = \sqrt[3]{x} + \frac{1}{\sqrt[3]{x}} \Rightarrow R_f = (-\infty, -2] \cup [2, +\infty)$ ✓

د) $y = \sqrt{x} + \frac{1}{\sqrt{x}} \Rightarrow R_f = [2, +\infty)$ ✓

ب

الف) $y = x^2 + \frac{1}{x^2 + 3} \Rightarrow y = x^2 + 3 \frac{1}{x^2 + 3} - 3 \Rightarrow R_f = [\frac{1}{3}, +\infty)$ ✓

ب) $x = \frac{x^2 + \epsilon}{\sqrt{x^2 + \epsilon}} \Rightarrow y = \frac{x^2 + \epsilon + 1}{\sqrt{x^2 + \epsilon}} \Rightarrow y = \sqrt{x^2 + \epsilon} + \frac{1}{\sqrt{x^2 + \epsilon}}$

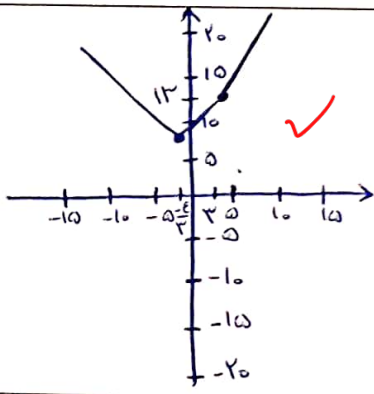
$\Rightarrow R_f = [\frac{1}{2}, +\infty)$ $\Rightarrow \min y = \sqrt{\epsilon} + \frac{1}{\sqrt{\epsilon}} = 2 + \frac{1}{2} \Rightarrow R_f = [\frac{5}{2}, +\infty)$ ✓

$\alpha = \sqrt{x^2 + \epsilon} = 1$ چون x متغیر

ج

$y = |x - 3| + |3x + 4|$

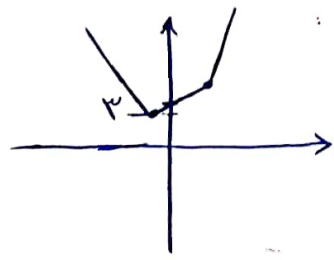
$\begin{cases} x - 3 + 3x + \epsilon & x > 3 \\ 3x + \epsilon - x + 3 & 3 \geq x \geq -\frac{\epsilon}{3} \\ -x + 3 - 3x - \epsilon & x < -\frac{\epsilon}{3} \end{cases} \Rightarrow \begin{cases} \epsilon x + 1 & x > 3 \\ 4x + 6 & 3 \geq x \geq -\frac{\epsilon}{3} \\ -\epsilon x + 1 & x < -\frac{\epsilon}{3} \end{cases}$



د

الف) $y = |x - 2| + |2x + 2|$

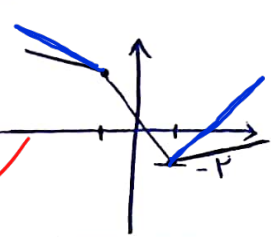
$\begin{cases} 3x & x > 2 \\ x + \epsilon & 2 \geq x \geq -1 \\ -2x & x < -1 \end{cases}$



$R_f = [2, +\infty)$ ✓

ب) $y = |x - 1| - |x + 1|$

$\begin{cases} x - 3 & x > 1 \\ -x + 1 & 1 \geq x \geq -1 \\ -x + 3 & x < -1 \end{cases}$



$R_f = [-2, +\infty)$ ✓

ه

الف) $y = x^3 - 3x^2 + 3x \Rightarrow y - 1 = x^3 - 3x^2 + 3x - 1 \Rightarrow y - 1 = (x - 1)^3 \Rightarrow \sqrt[3]{y - 1} = x - 1$
 $\Rightarrow x = \sqrt[3]{y - 1} + 1 \Rightarrow R_f = \mathbb{R}$ ✓

ب) $y = \frac{1}{x^2 - 2x} \Rightarrow yx^2 - 2yx - 1 = 0 \Rightarrow \Delta = 4y^2 + 4y \Rightarrow x = \frac{2y \pm \sqrt{4y^2 + 4y}}{2y}$
 $\Rightarrow y \neq 0, 4y^2 + 4y \geq 0 \Rightarrow y^2 + y \geq 0 \Rightarrow \frac{1}{y} \geq 0 \Rightarrow R_f = (-\infty, -1] \cup (0, +\infty)$ ✓

الف) $y = x^2 - 4x + 10 \xrightarrow{a > 0} \begin{cases} x_{\min} = \frac{-b}{2a} = 2 \\ y_{\min} = y_{\text{min}} = 6 \end{cases} \Rightarrow R_f = [6, +\infty)$ ✓

ب) $y = x^2 + 4x + 3 \xrightarrow{a < 0} \begin{cases} x_{\max} = \frac{-b}{2a} = -2 \\ y_{\max} = 1 \end{cases} \Rightarrow R_f = (-\infty, 1]$ ✓

ج) $y = \sqrt{x^2 - 4x - 2} \xrightarrow{a > 0} \begin{cases} x_{\min} = \frac{-b}{2a} = 2 \\ y_{\min} = -2 \end{cases} \Rightarrow R_f = \sqrt{[-2, +\infty)} = [0, +\infty)$ ✓

د) $y = \sqrt{4 - x^2} \xrightarrow{a < 0} \begin{cases} x_{\max} = \frac{-b}{2a} = 0 \\ y_{\max} = 2 \end{cases} \Rightarrow R_f = \sqrt{(-\infty, 4]} \Rightarrow R_f = [0, 2]$ ✓

الف) $y = x^3 - 6x^2 + 9x + 1 \quad R_f = \mathbb{R}$ ✓

ب) $y = x^3 - 9x^2 + 13x + 2 \quad R_f = \mathbb{R}$ ✓

ج) $y = \sqrt{x^3 - 9x^2 + 14x + 1} \quad R_f = [0, +\infty)$ ✓

د) $y = (x^3 - 6x^2 + 9x + 1)^2 \quad R_f = (\mathbb{R})^2 = [0, +\infty)$ ✓

الف) $y = \frac{x+1}{x+2} \quad R_f = \mathbb{R} - \left\{\frac{a}{c}\right\} = \mathbb{R} - \left\{\frac{1}{2}\right\}$ ✓

ب) $y = \frac{2x+1}{x+1} \quad R_f = \mathbb{R} - \left\{\frac{a}{c}\right\} = \mathbb{R} - \left\{\frac{1}{2}\right\}$ ✓

الف) $y = \sqrt{\frac{x+1}{x+2}} \Rightarrow R_f = \sqrt{\mathbb{R} - \left\{\frac{1}{2}\right\}} = [0, +\infty) - \left\{\sqrt{\frac{1}{2}}\right\}$ ✓

ب) $y = \sqrt{\frac{x+1}{x-2}} \Rightarrow R_f = \sqrt{\mathbb{R} - \left\{-\frac{1}{2}\right\}} = [0, +\infty)$ ✓