

الف) $y = x^3 - 3x^2 + 3x \Rightarrow y - 1 = x^3 - 3x^2 + 3x - 1$
 $\Rightarrow (x-1)^3 = y-1 \Rightarrow x-1 = \sqrt[3]{y-1} \Rightarrow x = \sqrt[3]{y-1} + 1 \Rightarrow R_y = \mathbb{R}$ ✓

ب) $y = \frac{1}{x^2 - 2x} \Rightarrow x^2 y - 2xy = 1 \Rightarrow x^2 - 2x = \frac{1}{y} \Rightarrow x^2 - 2x + 1 = \frac{1}{y} + 1 \Rightarrow (x-1)^2 = \frac{y+1}{y}$
 $\Rightarrow x-1 = \sqrt{\frac{y+1}{y}} \Rightarrow x-1 = \sqrt{\frac{y+1}{y}} \Rightarrow x = \sqrt{\frac{y+1}{y}} + 1 \Rightarrow \frac{y+1}{y} \geq 0$

$R_y = (-\infty, -1] \cup (0, +\infty)$ ✓

الف) $y = x^2 - 4x + 1$ ext | $\frac{y}{a}$ $R_y = [9, +\infty)$ ✓

ب) $y = -x^2 + 4x + 3$ ext | $\frac{y}{a}$ $R_y = (-\infty, 12]$ ✓

ج) $y = \sqrt{x^2 - 4x - 3}$ ext | $\frac{y}{a}$ $\Rightarrow R_y = [0, +\infty)$ ✓

د) $y = \sqrt{4x - x^2} \Rightarrow y = \sqrt{-x^2 + 4x}$ ext | $\frac{y}{a}$ $\Rightarrow R_y = [0, 2]$ ✓

الف) $y = x^2 - 5x + 1$ $\Rightarrow R_y = \mathbb{R}$ ✓

ب) $y = x^2 - 4x^2 + 3x + 2$ $\Rightarrow R_y = \mathbb{R}$ ✓

ج) $y = \sqrt{x^2 - 4x^2 + 5x + 1}$ $\Rightarrow R_y = [0, +\infty)$ ✓

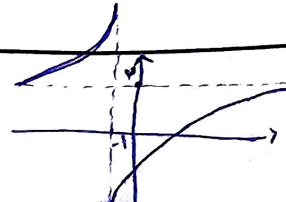
د) $y = (x^2 - 4x^2 + 3x + 1)$ $\Rightarrow R_y = [0, +\infty)$ ✓

الف) $y = \frac{3x+1}{x-2}$ $\frac{ad \neq bc}{\Rightarrow} R_y = \mathbb{R} - \{3\}$ ✓

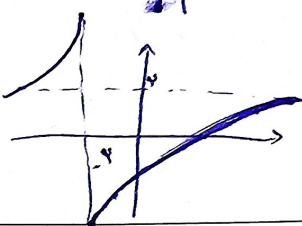
ب) $y = \frac{2x+1}{x+1}$ $\frac{ad \neq bc}{\Rightarrow} R_y = \mathbb{R} - \{2\}$ ✓

الف) $y = \sqrt{\frac{2x+1}{x+1}}$ $\Rightarrow R_y = [0, +\infty) - \{\sqrt{2}\}$ ✓

ب) $y = \sqrt{\frac{3x+1}{2-x}}$ $\Rightarrow R_y = [0, +\infty)$ ✓

الف) $y = \frac{2x-3}{x+1} \rightarrow \begin{cases} \text{مجاوب افقي} = \frac{a}{c} = 2 \\ \text{مجاوب قائم} = -1 \end{cases}$  (2)

$x \rightarrow \infty \Rightarrow y = 2$

ب) $y = \frac{2x-1}{x+2} \rightarrow \begin{cases} \text{مجاوب افقي} = 2 \\ \text{مجاوب قائم} = -2 \end{cases}$  6

$x \rightarrow \infty \Rightarrow y = 2$

الف) $y = \sin x + \frac{1}{\sin x} \Rightarrow R_y = (-\infty, -2] \cup [2, +\infty)$ ✓

ب) $y = x^3 + \frac{1}{x^3} \Rightarrow R_y = (-\infty, -2] \cup [2, +\infty)$ ✓

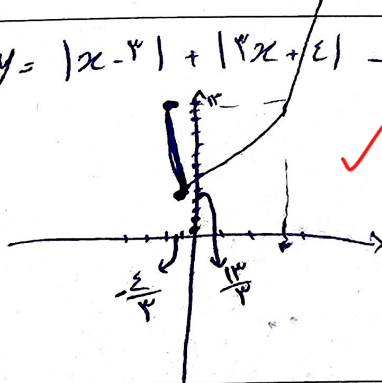
ج) $y = \sqrt[3]{x} + \frac{1}{\sqrt[3]{x}} \Rightarrow R_y = (-\infty, -2] \cup [2, +\infty)$ ✓

د) $y = \sqrt{x} + \frac{1}{\sqrt{x}} \Rightarrow R_y = [2, +\infty)$ ✓

الف) $y = x^2 + \frac{1}{x^2+3} \Rightarrow y = (x^2+3 + \frac{1}{x^2+3}) - 3$

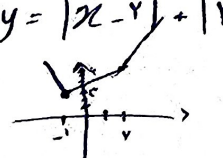
$x^2+3 \geq 3 \Rightarrow R_y = [3\frac{1}{4}, +\infty) = [\frac{13}{4}, +\infty) \Rightarrow R_y = [\frac{13}{4}, +\infty)$ (2)

ب) $y = \frac{x^2+5}{\sqrt{x^2+4}} = \frac{(x^2+4)+1}{\sqrt{x^2+4}} = \sqrt{x^2+4} + \frac{1}{\sqrt{x^2+4}} \Rightarrow R_y = [2\frac{1}{2}, +\infty) = [\frac{5}{2}, +\infty)$ ✓

$y = |x-3| + |3x+4| \rightarrow$  ✓

$\begin{array}{c} -\frac{4}{3} \qquad 3 \\ \hline -x+3-3x-4 \quad -x+3+3x+4 \quad x-3+3x+4 \\ = -4x-1 \quad = 2x+7 \quad = 4x+1 \\ \text{---} \quad \text{---} \quad \text{---} \\ \text{---} \quad \text{---} \quad \text{---} \end{array}$

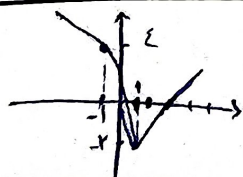
$R_y = [\frac{13}{3}, +\infty) \quad D_y = \mathbb{R}$

الف) $y = |x-2| + |2x+2| \rightarrow$  ✓

$\begin{array}{c} 1 \qquad 2 \\ \hline -x+2-2x-2 \quad -x+2+2x+2 \quad x-2+2x+2 \\ = -3x \quad = x+4 \quad = 3x \\ \text{---} \quad \text{---} \quad \text{---} \\ \text{---} \quad \text{---} \quad \text{---} \end{array}$

ب) $y = |2x-2| - |x+1| \rightarrow$

$\begin{array}{c} -1 \qquad 1 \\ \hline -2x+2-x-1 \quad -2x+2-x+1 \quad 2x-2-x-1 \\ = -3x+1 \quad = -3x+3 \quad = x-3 \\ \text{---} \quad \text{---} \quad \text{---} \\ \text{---} \quad \text{---} \quad \text{---} \end{array}$



$R_y = [-2, +\infty)$ ✓