

۱۷، ۱۷۵

نام و نام خانوادگی محمد نواز ضیائی پاسخنامه تشریحی تکلیف شماره کلاس پازسم B

$$y = \sqrt{x^3 f(x+1)} \Rightarrow f(x+1) = \left(\frac{1}{x}\right)^{(x+1)-2} \Rightarrow \left(\frac{1}{x}\right)^{x-1}$$

$$\Rightarrow x^3 f(x+1) \Rightarrow x^3 \left(\left(\frac{1}{x}\right)^{x-1} - 1\right) \Rightarrow x^3 \left(\left(\frac{1}{x}\right)^{x-1} - 1\right) \geq 0$$

$$\Rightarrow x^3 = 0 \Rightarrow x = 0 \quad \& \quad \left(\frac{1}{x}\right)^{x-1} - 1 = 0 \Rightarrow \left(\frac{1}{x}\right)^{x-1} = 1 \Rightarrow x = 1$$

$$\begin{array}{c|c|c|c|c|c} x & - & 0 & + & 1 & - \\ \hline f(x) & - & 0 & + & 0 & - \end{array} \Rightarrow Df = [0, 1] \checkmark$$

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$$f(x) = \sqrt{x+|x+2|} \Rightarrow f(-x) = \sqrt{-x+|2-x|} \Rightarrow -x+|2-x| \geq 0$$

$$\Rightarrow |2-x| \Rightarrow x \leq 2 \Rightarrow |2-x| = 2-x \Rightarrow -x+(2-x) = 2-2x$$

$$\Rightarrow 2-2x \geq 0 \Rightarrow x \leq 1 \quad \& \quad |2-x| \Rightarrow x > 2 \Rightarrow |2-x| = x-2 \Rightarrow x-2-x = -2$$

$$\Rightarrow -x+(x-2) = -2 \Rightarrow -2 = -2 \checkmark \text{ صحیح است}$$

$$Df(-x) \text{ or } Df = (-\infty, 1] \checkmark$$

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$$y = \sqrt{\frac{x-1}{f(x)}} \Rightarrow x-1=0 \Rightarrow x=1 \quad \& \quad f(x)=0 \Rightarrow x=2, 3, \dots$$

$$\begin{array}{c|c|c|c|c|c} x & - & 1 & + & 2 & 3 \\ \hline f(x) & + & 0 & - & - & 0 \end{array} \Rightarrow Df = [-1, 1] \cup (2, 3) \checkmark$$

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$$y_1 = \frac{1}{4x^2 - \sqrt{x^2 - 5x - 3}} \quad \& \quad y_2 = \frac{1}{4x^2 + \alpha x + b} \Rightarrow Df_{y_1} = Df_{y_2}$$

$$\Rightarrow y_1 = \frac{1}{(4x^2 - x - 3)(4x^2 + x + 1)} \Rightarrow 4x^2 + x + 1 \Rightarrow \Delta < 0 \Rightarrow \text{همیشه مثبت}$$

$$4x^2 - x - 3 = 4x^2 + \alpha x + b \Rightarrow \alpha = -1 \quad b = -3 \Rightarrow \sqrt{-(\alpha+b)} = \sqrt{-(-1-3)} = \sqrt{4} = \pm 2$$

ریشه دوم با جذر فرق دارد، از زیر رادیکال فقط + میاد

$$f(x) = x^3 + x \Rightarrow y = \sqrt{f(x) - f\left(\frac{x}{x}\right)} = (x^3 + x) - \left(\left(\frac{x}{x}\right)^3 + \frac{x}{x}\right)$$

$$\Rightarrow x^3 + x - \frac{x^3}{x^2} - \frac{x}{x} \Rightarrow x^3 \left(1 - \frac{1}{x}\right) + x \left(1 - \frac{1}{x}\right) \Rightarrow \frac{x^3}{x^2} x + \frac{x}{x} x$$

$$\Rightarrow \frac{x^3}{x^2} x + \frac{x}{x} x \Rightarrow x \left(\frac{x^3}{x^2} x + \frac{x}{x}\right) \Rightarrow x \left(\frac{4x^3 + 1}{x^2}\right) \Rightarrow x \left(\frac{4x^3 + 1}{x^2}\right)$$

$$y = \sqrt{\frac{x(4x^3 + 1)}{x^2}} \Rightarrow \sqrt{\frac{x(4x^3 + 1)}{x}} \Rightarrow \sqrt{4x^3 + 1} \quad \& \quad x=0 \Rightarrow Df = [0, \infty)$$

همیشه مثبت

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$$f(x+y) = 2xy - 1 \Rightarrow f(10) + f(1) = ?$$

$$\Rightarrow x+y = y \Rightarrow f(y) = 2xy - 1 \Rightarrow xy = 10 \Rightarrow x+y - 10 = 0 \Rightarrow x = 1 \Rightarrow 2(1)y - 1 = 10$$

$$f(1) \Rightarrow xy = 1 \Rightarrow xy - 1 = 0 \Rightarrow x = 1 \Rightarrow 2(1)y - 1 = 10 \Rightarrow y = 1$$

$$\rightarrow f(10) + f(1) = 10 + 1 = 11 \quad \checkmark$$

$$f(x) = x^3 - 9x^2 + 17x \Rightarrow (x-2)^3 = x^3 - 9x^2 + 17x - 8 \Rightarrow f(x) = (x-2)^3 + 8$$

$$g(x) = x^3 + 9x^2 + 17x \Rightarrow (x+2)^3 = x^3 + 9x^2 + 17x + 8 \Rightarrow g(x) = (x+2)^3 - 8$$

$$x = \sqrt[3]{10} - 2 \Rightarrow (\sqrt[3]{10} + 2 - 2)^3 - 8 = 10 - 8 = 2 \quad \checkmark$$

$$f(\sqrt[3]{10} + 2) = (\sqrt[3]{10} + 2 - 2)^3 + 8 = 10 + 8 = 18$$

$$\frac{g(\sqrt[3]{10} - 2)}{f(\sqrt[3]{10} + 2)} = \frac{2}{18} = \frac{1}{9} \quad \checkmark$$

$$f(x) = \sqrt{x + \varepsilon \sqrt{x - \varepsilon}} \quad g(x) = \sqrt{x - \varepsilon \sqrt{x - \varepsilon}} \quad a, \varepsilon \Rightarrow f(x) + g(x) = a + b \sqrt{x + c}$$

$$(f+g)^2 \Rightarrow f^2 + g^2 + 2fg \Rightarrow f^2 + g^2 = (x + \varepsilon \sqrt{x - \varepsilon}) + (x - \varepsilon \sqrt{x - \varepsilon}) = 2x$$

$$fg = \sqrt{(x + \varepsilon \sqrt{x - \varepsilon})(x - \varepsilon \sqrt{x - \varepsilon})} = \sqrt{x^2 - \varepsilon^2(x - \varepsilon)} = \sqrt{x^2 - \varepsilon x + \varepsilon^2} = x - \varepsilon \Rightarrow (f+g)^2 = 2x + 2(x - \varepsilon) = 4x - 2\varepsilon$$

$$f+g = \sqrt{4x - 2\varepsilon} \Rightarrow 2\sqrt{x - \varepsilon} \Rightarrow \sqrt{x - \varepsilon} = a + b\sqrt{x + c}$$

$$a=0 \quad b=2 \quad c=-\varepsilon \quad \frac{a+b}{c} = \frac{2}{-\varepsilon} = -\frac{2}{\varepsilon} \quad \checkmark$$

$$f(x) = \frac{x+2}{x^2 - \varepsilon x + 3} \quad g(x) = \{(1, 1), (1, 0), (2, 2), (0, 1)\} \quad \frac{g}{f} = ?$$

$$x^2 - \varepsilon x + 3 = 0 \Rightarrow (x-1)(x-2) \Rightarrow x=1 \quad \text{or} \quad x=2$$

$$x+2=0 \Rightarrow x=-2 \Rightarrow \frac{g}{f} = -2 = 0$$

$$\frac{g}{f} = \{(1, \frac{1}{-2}), (0, \frac{1}{3})\} \Rightarrow \frac{g}{f} = \{(1, -\frac{1}{2}), (0, \frac{1}{3})\} \quad \checkmark$$

$$1) f(x) = \{(1, 2), (\sqrt{3}, -1), (2, 2), (-\frac{1}{2}, 4)\} \quad \checkmark$$

$$2) f(x^2) = \{(1, 2), (\sqrt{3}, -1), (2, 2), (-\frac{1}{2}, 4), (-\sqrt{3}, -1)\} \quad \checkmark$$

$$3) f(g(x)+1) = \{(-2, 1), (1, 3), (3, 4), (-1, 1)\} \quad \checkmark$$

$$4) \frac{xf}{y} = \frac{\{(1, 4), (2, -2), (3, 4), (-1, 2)\}}{\{(1, 0), (1, -1), (2, 2), (-1, 0)\}} \Rightarrow \{(1, -4), (2, -1), (3, 4), (-1, 2)\} \quad \checkmark$$