

$$f_{(n+1)} = \frac{1}{p} \left(\frac{n+1}{p} - 1 \right) = \frac{1}{p} \left(\frac{n-1}{p} - 1 \right) \quad n^{\frac{p}{p-1}} f_{(n+1)} \geq 0 \rightarrow n^{\frac{p}{p-1}} \left(\frac{1}{p} - 1 \right) \geq 0$$

$$\frac{1}{p} - 1 \geq 0 \quad \frac{1}{p} - 1 = 1 \rightarrow n-1 = 0 \rightarrow n = 1$$

$$D_f = [0, 1]$$

1

$$f(-n) = \sqrt{(-n) + |n-2|} \quad |a| = |-a| \Rightarrow |-n+2| = |-(n-2)| \Rightarrow |n-2|$$

$$f(-n) = \sqrt{(-n) + |n-2|} \Rightarrow -n + |n-2| \geq 0 \quad \begin{cases} n \geq 2 \\ n \leq 2 \end{cases}$$

$$\begin{aligned} n \geq 2 & \rightarrow -n + n - 2 \geq 0 \rightarrow -2 \geq 0 \quad \times \\ n \leq 2 & \rightarrow -n - n + 2 \geq 0 \rightarrow -2n \geq -2 \rightarrow n \leq 1 \quad \checkmark \end{aligned}$$

$$D_{f(-n)} = (-\infty, 1]$$

2

$$\frac{n-1}{f(n)} \geq 0 \quad \begin{cases} \frac{+}{+} \rightarrow (2, 3) & 1 \\ \frac{0}{-} \rightarrow [2, 3] & 2 \\ \frac{-}{-} \rightarrow (-2, 1) & 3 \end{cases} \quad 1 \cup 2 \cup 3 = (-2, 1] \cup (2, 3)$$

3

$$\frac{1}{9a^2 - 4b^2 - 4c^2} = \frac{1}{4m^2 + 4n^2 + 4} \quad 9a^2 - 4b^2 - 4c^2 = (3a^2 + 4b^2 + 4c^2) \left(\frac{3a^2}{3a^2 + 4b^2 + 4c^2} - 1 \right)$$

$$\Delta_1 = b^2 - 4ac = 1 - 12 = -11 \quad \Delta_2 = 1 + 12 = 13$$

$$\frac{1 \pm \sqrt{13}}{2} = \frac{-a \pm \sqrt{a^2 - 4b^2}}{2} \Rightarrow 1 \pm \sqrt{13} = -a \pm \sqrt{a^2 - 12b^2}$$

$$a^2 - 12b^2 = 10 \quad 1 - 12b^2 = 10 \quad -12b^2 = 9 \quad b \leq \frac{3}{2} \quad \sqrt{-(1+12)} = \sqrt{13}$$

4

$$f(m) - f\left(\frac{c}{n}\right) \geq 0 \quad n^{\frac{p}{p-1}} + m - \left(\frac{c}{n}\right)^{\frac{p}{p-1}} + \frac{c}{n} \geq 0 \rightarrow \frac{n^{\frac{p}{p-1}} + m^{\frac{p}{p-1}} - \left(\frac{c}{n}\right)^{\frac{p}{p-1}}}{n^{\frac{p}{p-1}}} \geq 0$$

$$(n^{\frac{p}{p-1}} - \left(\frac{c}{n}\right)^{\frac{p}{p-1}}) + (m^{\frac{p}{p-1}} - \left(\frac{c}{n}\right)^{\frac{p}{p-1}}) \rightarrow (n^{\frac{p}{p-1}} - 1)(n^{\frac{p}{p-1}} + 1) + m^{\frac{p}{p-1}}(n^{\frac{p}{p-1}} - 1) \geq 0$$

$$(n^{\frac{p}{p-1}} - 1)(n^{\frac{p}{p-1}} + m^{\frac{p}{p-1}} + 1) \geq 0 \rightarrow \frac{(n^{\frac{p}{p-1}} - 1)}{n^{\frac{p}{p-1}}} \geq 0 \quad \begin{aligned} & -\frac{1}{n} + \frac{1}{n} - \frac{1}{n} \rightarrow [-1, 0) \cup [1, \infty) \end{aligned}$$

$x^{10} + x > 1 \rightarrow x(x^9 + 1) > 1 \rightarrow x = 1$ $x^5 + x = 1 \rightarrow x(x^4 + 1) = 1 \rightarrow x = 1$ $x^4 - 1 + x^3 - 1 = (x-1) + (x-1) = \textcircled{V}$	6
$x^5 - \frac{1}{2}x^4 + 12x = (x-1)^5 + 1 \quad x^5 + 9x^4 + 12x = (x+1)^5 - 12x$ $f(\sqrt[5]{x+1}) = ((\sqrt[5]{x+1}) - 1)^5 + 1 = (\sqrt[5]{x+1})^5 + 1 = x+1+1 = 2$ $2(\sqrt[5]{x-1}) = ((\sqrt[5]{x-1}) + 1)^5 - 12x = (\sqrt[5]{x-1})^5 - 12x = x-1-12x = -11x-1 \leq 10$ $\frac{2(\sqrt[5]{x-1})}{f(\sqrt[5]{x+1})} = \frac{-11x-1}{2} \leq -1$	7
$x \leq u - t + t \Rightarrow f(u) = \sqrt{(u-t)^2 + t^2} + t \sqrt{u-t} \quad \sqrt{u-t} = u$ $f(u) = \sqrt{u^2 + t^2} + t u = \sqrt{(u+t)^2} \Rightarrow u+t = \sqrt{u^2 + t^2} + t$ $2(u) = \sqrt{(u-t)^2 + t^2} + t \sqrt{u-t} = \sqrt{u^2 - 2ut + t^2 + t^2} + t \sqrt{u-t} = \sqrt{u^2 - 2ut + 2t^2} + t \sqrt{u-t}$ $2(u) \begin{cases} t \leq u & \rightarrow \sqrt{u-t} = \sqrt{u-t} + t - t \sqrt{u-t} = t \sqrt{u-t} + t \sqrt{u-t} = 2t \sqrt{u-t} \\ u \geq t & \rightarrow \sqrt{u-t} = \sqrt{u-t} + t - t \sqrt{u-t} = t \sqrt{u-t} + t \sqrt{u-t} = 2t \sqrt{u-t} \end{cases}$	8
$x^5 - 12x + 3 = (x-1)(x-12) \Rightarrow f_1 \neq f_2 = 0$ $f(x) = \frac{x+1}{x^2 - f(x) + 1} > -1 \quad \frac{2(x)}{f(x)} = \frac{1}{-f} \quad f_0 = \frac{x+1}{x^2 - f(x) + 1} = \frac{1}{x} \quad \frac{2}{f_0} = \frac{1}{x} > 5$ $\frac{2}{f} = \left\{ \left(1, \frac{1}{f}\right), (0, 5) \right\}$	9
1) $\left\{ \left(\frac{1}{2}, 2\right), \left(\frac{5}{2}, 1\right), (2, 2), \left(-\frac{1}{2}, 4\right) \right\}$ 2) $\left\{ (1, 2), (-1, 2), (\sqrt{2}, -1), (-\sqrt{2}, -1), (2, 2), (-2, 2) \right\}$ 3) $\left\{ (-2, 1), (1, 2), (2, 1), (-1, 1) \right\}$ 4) $\left\{ (1, -1), (3, -1) \right\}$	10