بنیامین نوروزی - یازدهم پسر B - تکلیف شمارهٔ ۵

۱)

$x^3 f(x+1) \geq 0$

$f(x+1) = (\frac{1}{2})^{x+1-2} - 1$

$x^3\left((\frac{1}{2})^{x-1} - 1\right) \geq 0 \Rightarrow (\frac{1}{2})^{x-1} = 1 \Rightarrow x-1=0 \Rightarrow \boxed{x=1}$

$x^3 = 0 \Rightarrow \boxed{x=0}$

$\begin{array}{ccccc} & 0 & & 1 & \\ - & \phi & + & \phi & - \end{array} \Rightarrow D_f = [0,1]$

۲)

$f(-x) = \sqrt{-x+|-x+2|}$

$x>2 \rightarrow -x+x-2 = -2$

$x=2 \rightarrow (-x) \geq 0 \Rightarrow x \leq 0$ (I)

$x<2 \rightarrow -x-x+2 = -2x+2 \geq 0 \Rightarrow x \leq 1$ (II)

$D_f = (-\infty, 1] = I \cap II$

۳)

$\frac{x-1}{f(x)} \geq 0$

(جاهایی که در نمودار $y=0$) $\Rightarrow x = -4, 2, 3$

$\begin{array}{ccccccccc} & -4 & & 1 & & 2 & & 3 & \\ - & \text{ت.ن} & + & \phi & - & \text{ت.ن} & + & \text{ت.ن} & - \end{array} \Rightarrow D_f = (-4,1] \cup (2,3)$

۴)

$9x^4 - 7x^2 - 4x - 3 = (9x^4 - 6x^2 + 1) - (x^2 + 4x + 4) = (3x^2-1)^2 = (x+2)^2$

$\Rightarrow 3x^2 - 1 - x - 2 = 0 \Rightarrow 3x^2 - x - 3 = 3x^2 + ax + b$

$a = -1$, $b = -3$

$\Rightarrow \sqrt{-(-4)} = 2$

۵)

$f(x) - f(\frac{4}{x}) \geq 0 \Rightarrow x^3 + x - \left((\frac{4}{x})^3 + \frac{4}{x}\right) \geq 0 \Rightarrow x^3 + x - \left(\frac{64 + 4x^2}{x^3}\right) \Rightarrow \frac{x^6 + x^4 - 4x^2 - 64}{x^3} \geq 0$

اگر $x^2 = t$ $\Rightarrow$ صورت $= t^3 + t^2 - 4t - 64 = 0$ ($t=4$) $\Rightarrow 64 + 16 - 16 - 64 = 0 \Rightarrow \boxed{x = \pm 2}$

$\boxed{x=0 \text{ اگر } \rightarrow \text{مخرج}=0}$

$\begin{array}{ccccccc} & -2 & & 0 & & +2 & \\ - & \phi & + & \text{ت.ن} & - & \phi & + \end{array} \Rightarrow D_f = [-2, 0) \cup [2, +\infty)$

۶)

$x^3 + x = 10 \Rightarrow x^3 + x - 10 = 0$ ($x=2$) $\rightarrow f(10) = 8 - 1 = 7$ ($x=2$)

$x^3 + x = 2 \Rightarrow x = 1 \rightarrow f(2) = 2 - 1 = 1$

$f(2) + f(10) = 8$

(۷)

$f(x) = x^3 - 6x(x-2) \xrightarrow{x=\sqrt[3]{2}+2} 2+8+6\sqrt[3]{4} + 12\sqrt[3]{2} + (-6\sqrt[3]{2} - 12)(\sqrt[3]{2}) = 10$

$g(x) = x^3 + 9x^2 + 27x \xrightarrow{x=\sqrt[3]{2}-3} (8-27) - 9\sqrt[3]{4} + 27\sqrt[3]{2} + 9\sqrt[3]{4} + (-18) + 27\sqrt[3]{2} + (18) - 54\sqrt[3]{2} = -28$

$\Rightarrow \frac{g(\sqrt[3]{2}-3)}{f(\sqrt[3]{2}+2)} = (-2)$

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(۱)

$f(x) + g(x) = \sqrt{(x-4) - 4\sqrt{x-5} + 4} + \sqrt{(x-4) + 4\sqrt{x-5} + 4} - \sqrt{(\sqrt{x-5}-2)^2}$

$+ \sqrt{(\sqrt{x-5}+2)^2} = |\sqrt{x-5}+2| + |\sqrt{x-5}-2| \Rightarrow 2\sqrt{x-5} = a + b\sqrt{x} + c$

$a=0 \;/\; b=2 \;/\; c=-5 \Rightarrow \frac{a+b}{c} = \frac{-1}{2}$

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(۹)

$\frac{g}{f} = \{g(x) \cap f(x)\} - \{x \mid f(x)=0\}$

$\frac{x+2}{x^2-5x+4} = \frac{x+2}{(x-4)(x-1)}$ ④ ①

$x=2 \Rightarrow f(2) = \frac{4}{-1} = (-4)$

$x=0 \Rightarrow f(0) = \frac{2}{4}$

$\Rightarrow \{(2,1),(1,0),(3,5),(0,4)\}$

$\Rightarrow \frac{g}{f} = \{(2,\frac{-1}{4}),(0,6)\}$

$\Rightarrow \frac{g}{f} = \{(2,\frac{1}{-4}),(0,\frac{2}{3})\}$

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(۱۰)

الف) $f(2x) = \{(\frac{1}{2},2),(\frac{3}{2},-1),(2,2),(\frac{-1}{2},6)\}$

ب) $f(x^2) = \{(1,2),(\sqrt{3},-1),(2,2),(-1,2),(-\sqrt{3},-1),(-2,2)\}$

ج) $2g^2(x)+1 = \{(-2,19),(1,3),(3,9),(9,3),(1,-1),(-1,1)\}$

د) $\frac{f^2}{g} = \{(1,-4),(3,-1),(-1,\frac{12}{5})\}$

«خسته نباشید»