

1) $y = \sqrt{x^3 f(x+1)}$ و $f(x) = \frac{1}{x} x - 1$

$\rightarrow x^3 f(x+1) \geq 0$
 $\rightarrow f(x+1) \geq \frac{1}{x} x - 1$

$\Rightarrow x^3 (\frac{1}{x+1} - 1) \geq 0$
 $\rightarrow \frac{0}{-1+1} \rightarrow D_f = [0, 1]$

2) $f(x) = \sqrt{x + |x+2|}$

$\rightarrow f(-x) = \sqrt{-x + |-x+2|} \rightarrow |-x+2| \geq x \xrightarrow{\text{توانیم برابری کنیم}} x^2 - 4x \geq x^2$
 $\rightarrow -4x \geq 0 \rightarrow x \leq 0 \rightarrow D = (-\infty, 0]$

3) $y = \sqrt{\frac{x-1}{f(x)}}$

$\rightarrow$

$x-1$	-	-	0	+	+	+
$f(x)$	+	0	-	-	0	+
	-	0	+	+	0	-

$\rightarrow D_f = (-\infty, -1] \cup (2, 3)$

4) $y_1 = \frac{1}{4x^2 - 4x^2(x-3)} = \frac{1}{(4x^2 - 4x^2 + 1) - (x^2 + 4x + 4)} = \frac{1}{(3x^2 - 1) - (x+2)^2}$

$= \frac{1}{(3x^2 + x + 1)(3x^2 - x - 3)}$

$\Rightarrow D_{y_1} = \mathbb{R} - \{3x^2 - x - 3\}$

$= D_{y_2} = \mathbb{R} - \{3x^2 + ax + b\}$

$\Rightarrow 3x^2 - x - 3 = 3x^2 + ax + b \Rightarrow \begin{cases} a = -1 \\ b = -3 \end{cases}$
 $\Rightarrow \sqrt{-(-1)} = \sqrt{1} = 1$

5) $\sqrt{f(x) + f(\frac{1}{x})} = \sqrt{x^3 + x - (\frac{1}{x})^3 - (\frac{1}{x})} = \sqrt{(x - \frac{1}{x})(x^2 + \frac{1}{x} + \frac{1}{x^2}) + (x - \frac{1}{x})}$

$\rightarrow x \geq \frac{1}{x} \rightarrow x^2 \geq 1 \rightarrow x \geq 1$
 $\rightarrow \frac{-1}{-1+1-1} = \frac{-1}{-1} = 1$
 $D = [-1, 0) \cup [1, +\infty)$

9) ~~$f(x) + f(1/x)$~~

$$f(x) = f(x^3 + x) \rightarrow x \geq 1 \rightarrow x^3 - 1 \geq x(1) - 1 \geq 0 \quad (2)$$

$$f(1/x) = f(x^3 + x) \rightarrow x \leq 1 \rightarrow x(x^3) - 1 \geq 1 - 1 \geq 0 \quad (2)$$

$$\rightarrow f(x) + f(1/x) \geq 1 + 1 \geq 2 \quad (1) \quad \checkmark$$

10) $f(x) = x^3 - 9x^2 + 12x \rightarrow \frac{g(\sqrt[3]{x} - 3)}{f(\sqrt[3]{x} + 2)}$

$g(x) = x^3 + 9x^2 + 12x$

$$\rightarrow \frac{x(x^2 + 9x + 12)}{x(x^2 - 9x + 12)} = \frac{(\sqrt[3]{x} - 3)((\sqrt[3]{x} - 3)^2 + 9(\sqrt[3]{x} - 3) + 12)}{(\sqrt[3]{x} + 2)((\sqrt[3]{x} + 2)^2 - 9(\sqrt[3]{x} + 2) + 12)}$$

$$= \frac{(\sqrt[3]{x} - 3)(\sqrt[3]{x}^2 + 9 - 9\sqrt[3]{x} + 9\sqrt[3]{x} - 27 + 12)}{(\sqrt[3]{x} + 2)(\sqrt[3]{x}^2 + 4 + 4\sqrt[3]{x} - 9\sqrt[3]{x} - 12 + 12)}$$

$$= \frac{(\sqrt[3]{x} - 3)(\sqrt[3]{x}^2 + 3\sqrt[3]{x} + 9)}{(\sqrt[3]{x} + 2)(\sqrt[3]{x}^2 - 2\sqrt[3]{x} + 4)}$$

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$$= \frac{x - 27}{x + 8} = \frac{-19}{19} = -1 \quad \checkmark$$

11) $f(x) = \sqrt{x + 4\sqrt{x-4}}$, $g(x) = \sqrt{x - 4\sqrt{x-4}}$, $x \geq 4$

$$\rightarrow f(x) + g(x) = (\sqrt{x + 4\sqrt{x-4}} + \sqrt{x - 4\sqrt{x-4}})^2 = x + 2\sqrt{x^2 - 16(x-4)}$$

$$= x + 2\sqrt{x^2 - 4x + 16} = x + 2\sqrt{(x-2)^2 + 12} = \sqrt{x + 2\sqrt{x-4}}$$

$$\rightarrow f + g = \sqrt{x + 2\sqrt{x-4}} = a + b\sqrt{x-4}$$

$$\rightarrow \sqrt{x-4} \leq 1 \rightarrow \sqrt{x+2\sqrt{x-4}} = \sqrt{14} = \sqrt{14} = \sqrt{14}$$

$$\rightarrow x \geq 4 \rightarrow \sqrt{x+2\sqrt{x-4}} = \sqrt{4x-4} = \sqrt{4(x-1)} = 2\sqrt{x-1}$$

$$\rightarrow \sqrt{x-4} = a + b\sqrt{x-4} \rightarrow \frac{b}{a} = \frac{1}{2} \rightarrow \frac{1+0}{-2} = \pm \frac{1}{2} = \left(-\frac{1}{2}\right) \quad \checkmark$$

12) $f(x) = \frac{x+2}{x^2 - 4x + 4} \rightarrow D_f = \mathbb{R} - \{2, 1\}$

$g(x) = \{(2, 1), (1, 0), (0, 2), (2, 4)\}$

$$\rightarrow \frac{g}{f} = \left\{ \left(2, \frac{1}{2}\right), \left(0, \frac{4}{2}\right) \right\} \quad \checkmark$$

$$f(x) = \{(1, 2), (3, -1), (4, 2), (-1, 4)\}$$

$$g(x) = \{(-2, 3), (1, -1), (3, 2), (4, 0)\}$$

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$$الف) f(g(x)) = \left\{ \left(\frac{1}{2}, 2 \right), (1, 2), (2, 2), \left(-\frac{1}{2}, 4 \right) \right\}$$

$$ب) f(x^2) = \{ (1, 2), (\sqrt{3}, -1), (\sqrt{2}, 2) \}$$

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$$ج) g^2(x+1) = \{ (-2, 1), (4, 3), (3, 4), (-1, 4) \}$$

$$د) \frac{f}{g} = \left\{ (1, -4), (3, -1), \left(-\frac{1}{2}, \frac{4}{0} \right) \right\}$$