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نام و نام خانوادگی ...... پاسخنامه تشریحی تکلیف شماره ۵ ...... کلاس ...... B

۱ (۲)

$f(x) = (\frac{1}{2})^{x-2} - 1 \quad f(x+1) = \frac{1}{2}^{x-1} - 1$

$y = \sqrt{x^3 f(x+1)} \rightarrow y = \sqrt{x^3(\frac{1}{2}^{x-1} - 1)} \geq 0 \rightarrow x^3(\frac{1}{2}^{x-1} - 1) \geq 0$

$\frac{1}{2}^{x-1} = 1$

$x - 1 = 0 \rightarrow x = 1$

$D_f = [0, 1]$

۲ (۲)

$f(x) = \sqrt{x + |x+2|} \rightarrow f(-x) = \sqrt{-x + |-x+2|}$

$-x + |-x+2| \geq 0$

$x \leq 2 \rightarrow -x - x + 2 \geq 0 \quad -2x + 2 \geq 0 \quad 2x \leq 2 \quad x \leq 1$

$x > 2 \rightarrow -x + x - 2 \geq 0 \quad -2 \geq 0 \; \times \; \emptyset$

$x \leq 2, \; x \leq 1$

$D_f = (-\infty, 1]$

۳ (۲)

$y = \sqrt{\frac{x-1}{f(x)}} \rightarrow x - 1 = 0 \quad x = 1 \quad \frac{x-1}{f(x)} \geq 0$

$f(x) = 0 \rightarrow x = 3, 2, -4$

$D_f = (-4, 1] \cup (2, 3)$

۴ (۲)

$y_1 = \frac{1}{9x^4 - 7x^2 + 3x - 3} \quad , \quad y_2 = \frac{1}{3x^2 + ax + b} \quad D_{y_1} = D_{y_2}$

$b = -3 \quad a = -1$

$3x^2 - x - 3 = 3x^2 + ax + b$

$y_1 = \frac{1}{9x^4 + 3x^3 - 3x^3 + 3x^2 - x^2 - 9x^2 - 4x - 3 = 0} \rightarrow (3x^2 + x + 1)(3x^2 - x - 3) = 0$

$\Delta = 1 - 12 = -11 \rightarrow \Delta < 0$ صفر نمی‌شود

این عبارت دامنه ندارد

$9x^4 + 3x^3 - 3x^3 + 3x^2 - x^2 - 9x^2 - x - 3x - 3 = 0 \rightarrow 3x^2(3x^2 + x + 1) - x(3x^2 + x + 1) - 3(3x^2 + x + 1) = 0$

۵ (۲)

$f(x) = x^3 + x \quad f(\frac{4}{x}) = (\frac{4}{x})^3 + \frac{4}{x}$

$D_f = [-2, 0) \cup (2, +\infty)$

$y = \sqrt{x^3 + x - (\frac{4}{x})^3 - \frac{4}{x}} \geq 0$

$1 = \frac{4}{x^2} \rightarrow x^2 = 4 \quad x = \pm 2$

$x - \frac{4}{x} \geq 0 \rightarrow x(1 - \frac{4}{x^2}) \geq 0$

$x^3 - (\frac{4}{x})^3 + x - \frac{4}{x} \rightarrow (x - \frac{4}{x})(x^2 + \frac{16}{x^2} + 4) + x - \frac{4}{x} \geq 0 \quad (x - \frac{4}{x})(x^2 + \frac{16}{x^2} + 4 + 1) \geq 0$

عبارت دوم اتحاد چاق و لاغر هیچگاه صفر نمی‌شود.

$f(x^3+x) = 2x^2 - 1 \to x=1 \to f(1+1) = 2-1$, $f(2) = 1$ ۲

$x=2 \to f(8+2) = 8-1$, $f(10) = 7$ ۶

$f(2)=? \to f(2)=1$

$f(10)=? \to f(10)=7$

$f(2)+f(10) \Rightarrow \boxed{1+7=8}$

$f(x) = x^3 - 6x^2 + 12x - 8 + 8 \to f(x) = (x-2)^3 + 8$ ۲

$g(x) = x^3 + 9x^2 + 27x + 27 - 27 \to g(x) = (x+3)^3 - 27$

$f(x) \xrightarrow{x=\sqrt[3]{2}+2} = (\sqrt[3]{2}+2-2)^3 + 8 = 10$

$g(x) = (x+3)^3 - 27 \xrightarrow{x=\sqrt[3]{7}-3} (\sqrt[3]{7}-3+3)^3 - 27 = 7 - 27 = -20$

$\frac{g(\sqrt[3]{7}-3)}{f(\sqrt[3]{2}+2)} = \frac{-20}{10} = \boxed{-2}$ ۷

$f(x)+g(x) = a+b\sqrt{x+c}$

$\underbrace{\sqrt{x-4\sqrt{x-4}} + \sqrt{x+4\sqrt{x-4}}}_{A^2}$ (A) $= a+b\sqrt{x+c}$

$A^2 = 4x-16$

$A = \sqrt{4x-16} \Rightarrow A = \sqrt{4(x-4)}$

$A = 2\sqrt{x-4}$

$2\sqrt{x-4} = a+b\sqrt{x+c} \to a+b = 2$, $c = -4$ ۸ ۲

$\boxed{\frac{a+b}{c} = \frac{2}{-4} = -\frac{1}{2}}$

$(\sqrt{x+4\sqrt{x-4}} + \sqrt{x-4\sqrt{x-4}})^2 = x + \cancel{4\sqrt{x-4}} + x - \cancel{4\sqrt{x-4}} + 2\sqrt{(x+4\sqrt{x-4})(x-4\sqrt{x-4})}$

$2x + 2\sqrt{(x+4\sqrt{x-4})(x-4\sqrt{x-4})} = 2x + 2\sqrt{x^2-16(x-4)} = 2x + 2\sqrt{x^2-16x+64} = 2x + 2\sqrt{(x-8)^2}$

مزدوج

$2x + 2x - 16 = \boxed{4x-16}$

$f(x) = \frac{x+2}{x^2-4x+3} \to D_f: \frac{x+2}{(x-1)(x-3)} \xrightarrow{\neq 0}$, $x = 1, 3$, $D_f = \mathbb{R} - \{1, 3\}$ ۲

$g(x) = \{(2,1), (1,-1), (3,0), (0,4)\}$, $D_g = \{2, 1, 3, 0\}$ ۹

$D_f \cap D_g = \{2, 0\}$

$\frac{g}{f} = \{(2, \frac{1}{-4}), (0, \frac{4}{\frac{2}{3}})\} \to \boxed{\{(2, -\frac{1}{4}), (0, 6)\}}$

$f(x) = \{(1,2), (3,-1), (4,2), (-1,6)\}$, $g(x) = \{(2,3), (1,-1), (3,2), (-1,0)\}$ ۲

الف) $f(2x) = \{(\frac{1}{2},2), (\frac{3}{2},-1), (2,2), (-\frac{1}{2},6)\}$

$2x=1$, $x=\frac{1}{2}$; $2x=-1$, $x=-\frac{1}{2}$; $2x=3$, $x=\frac{3}{2}$; $2x=4$, $x=2$

ب) $f(x^2) = \{(1,2), (-1,2), (\sqrt{3},-1), (-\sqrt{3},-1), (2,2), (-2,2)\}$

$x^2=1$, $x=\pm 1$; $x^2=3$, $x=\pm\sqrt{3}$; $x^2=4$, $x=\pm 2$; $x^2=-1$ ✗

دقت!

ج) $2g^2(x)+1 = \{(\overset{-2}{\cancel{2}}, 19), (1,3), (3,9), (-1,1)\}$

$2 \to 3 \to (3)^2 = 9 \xrightarrow{\times 2} 18 \xrightarrow{+1} 19$; $1 \to -1 \to 1 \xrightarrow{\times 2} 2 \xrightarrow{+1} 3$; $3 \to 2 \to 4 \xrightarrow{\times 2} 8 \xrightarrow{+1} 9$; $-1 \to 0 \to 0 \to 0 \to 1$

د) $\frac{f}{g} = \{(1, \frac{2}{-1}), (3, \frac{-1}{2}), \cancel{(-1, \frac{6}{0})}\}$

$D_f \cap D_g = \{1, 3, -1\}$

$\frac{f}{g} = \{(1,-2), (3,-\frac{1}{2})\}$