

نام و نام خانوادگی: ... (نام و نام خانوادگی) ... پاسخنامه تشریحی تکلیف شماره ... کلاس: ...

$$y = \sqrt{x^3 f(x+1)} = \sqrt{x^3 \left(\left(\frac{1}{x} \right)^{x-1} - 1 \right)} \geq 0 \Rightarrow \left(\frac{1}{x} \right)^{x-1} - 1 \geq 0$$

$$x < 1 \text{ (۱)} \text{ و } x^3 > 0 \text{ (۲)} \xrightarrow{\text{(۱) و (۲)}} D_f = [0, 1] \checkmark$$

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$$f(-x) = \sqrt{-x + |-x + 2|} = \sqrt{-x + |x - 2|} \geq 0$$

غیر مجاز $\xrightarrow{x \leq 2} |x - 2| = x - 2 \Rightarrow -x + (x - 2) = -2 < 0 \rightarrow$

$\xrightarrow{x < 2} |x - 2| = 2 - x \Rightarrow -x + (2 - x) = 2 - 2x \geq 0 \Rightarrow x \leq 1$

$$D_f = (-\infty, 1] \checkmark$$

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$$y = \sqrt{\frac{x-1}{f(x)}} \Rightarrow \frac{x-1}{f(x)} \geq 0 \text{ و } f(x) \neq 0 \Rightarrow x \neq -4 \text{ و } 2 \text{ و } 3 \rightarrow$$

$x-1 \rightarrow \begin{array}{c|c|c|c|} -4 & 1 & 2 & 3 \\ \hline + & - & + & - \end{array} \text{ (۱)}$

$\Rightarrow D_f = (2, 3) \cup (-4, 1] \checkmark$

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$D_{y_1} = D_{y_2}$ پس هر دو باید در یک نقطه یا نقاط مشابهی صفر شوند

$$\Rightarrow 3x^2 + ax + b \text{ باید با یکی از آنها برابر باشد} \Rightarrow 3x^2 + x + 1 \Rightarrow \sqrt{-(a+b)} = \sqrt{-2}x$$

$$\Rightarrow 3x^2 - x - 3 \Rightarrow \sqrt{-(a+b)} = \sqrt{4} = 2 \checkmark$$

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$$f(x) - f\left(\frac{1}{x}\right) = x^3 + x - \left(\frac{1}{x^3} + \frac{1}{x}\right) = \frac{x^6 + x^4 - 1 - x^2}{x^3} = \frac{(x^2-1)(x^2+1)(x^2+x+1)}{x^3}$$

$x^4 + 5x^2 + 14$ (مشتق دوم)

$x \neq 0$ و $\begin{array}{c|c|c|c|} -2 & 0 & 2 & 3 \\ \hline - & + & - & + \end{array} \Rightarrow D_f = [-2, 0) \cup [2, +\infty) \checkmark$

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$$f(2) = f(1^3 + 1) \Rightarrow f(2) = 2(1)^2 + (-1) = 1$$

$$f(10) = f(2^3 + 2) \Rightarrow f(10) = 2(2)^2 - 1 = 7$$

$$f(2) + f(10) = 1 + 7 = 8 \quad \checkmark$$

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$$f(x) = x^3 - 4x^2 + 12x \Rightarrow (x-2)^3 + 1$$

$$g(x) = x^3 + 9x^2 + 27x = (x+3)^3 - 27$$

$$\left. \begin{aligned} g(\sqrt[3]{27} - 3) &= (\sqrt[3]{27} - 3 + 3)^3 - 27 = -27 \\ f(\sqrt[3]{27} + 2) &= (\sqrt[3]{27} + 2 - 2)^3 + 1 = 1 \end{aligned} \right\} \Rightarrow \frac{-27}{1} = -27 \quad \checkmark$$

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$$f(x) + g(x) = \sqrt{x+4}\sqrt{x-4} + \sqrt{x-4}\sqrt{x+4} \xrightarrow{\text{بفرض}} t = \sqrt{x-4} \Rightarrow x = t^2 + 4$$

$$\Rightarrow \left. \begin{aligned} f(x) &= \sqrt{(t^2+4)+4}t = \sqrt{(t+2)^2} = t+2 \\ g(x) &= \sqrt{(t^2+4)-4}t = \sqrt{(t-2)^2} = |t-2| \end{aligned} \right\} f(x) + g(x) = t+2 + |t-2|$$

$$t \geq 2 \Rightarrow t+2+t-2=2t=2\sqrt{x-4} \Rightarrow a+b=2, c=4 \Rightarrow \frac{2}{4} = \frac{1}{2} \quad \checkmark$$

$$t < 2 \Rightarrow t+2-t+2=4 \rightarrow 4 = 4 + 0(\sqrt{x-4}) \quad \times$$

هر مقدار برای هر کد C می توان نوشت

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$$x=2 \Rightarrow g(2)=1, f(2) = \frac{2+2}{(2-1)(2-3)} = -4 \Rightarrow \frac{g}{f} = \frac{-1}{4}$$

$$x=1 \Rightarrow f(1) = \text{تعریف نشده}$$

$$x=3 \Rightarrow f(3) = \text{تعریف نشده}$$

$$x=0 \Rightarrow g(0)=4 \Rightarrow f(x) \Rightarrow f(0) = \frac{0+2}{(0-1)(0-3)} = \frac{2}{3} \Rightarrow \frac{g}{f} = \frac{4}{\frac{2}{3}} = 6$$

$$\frac{g}{f} = \left\{ \left(2, \frac{1}{4}\right), (0, 6) \right\} \quad \checkmark$$

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$$f(2x) = \left\{ \left(\frac{1}{2}, 2\right), \left(\frac{3}{2}, -1\right), (2, 2), \left(\frac{5}{2}, 4\right) \right\} \quad \checkmark$$

$$f(x^2) = \left\{ (\pm 1, 2), (\pm \sqrt{3}, -1), (\pm 2, 2) \right\} \quad \checkmark$$

$$2g(x)+1 = \left\{ (-2, 9), (1, 3), (3, 9), (-1, 1) \right\} \quad \checkmark$$

$$\frac{2f}{g} = \left\{ (1, -4), (3, -1) \right\} \quad \checkmark$$

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