

$f(x) = \left(\frac{1}{x}\right)^{x-1} \Rightarrow f(x+1) = \left(\frac{1}{x+1}\right)^{x-1} - 1$
 $y = \sqrt{x^x f(x+1)} \Rightarrow x^x f(x+1) \geq 0 \Rightarrow x^x \times (x^{1-x} - 1) \geq 0$
 $\frac{0}{-} \quad \frac{1}{+} \quad \frac{-}{-}$
 $D_f = [0, 1]$ ✓

$f(x) = \sqrt{x + (x+2)} \Rightarrow f(-x) = \sqrt{-x + (-x+2)} \Rightarrow -x + (-x+2) \geq 0$
 $(-x+2) = (x-2)$
 $\Rightarrow -x + (x-2) \geq 0 \Rightarrow \frac{2}{-2x+2} \geq 0 \Rightarrow \frac{-2}{-2x+2} \geq 0$
 $\frac{-2}{-2x+2} \geq 0 \Rightarrow \frac{-2}{-2(x-1)} \geq 0 \Rightarrow \frac{1}{x-1} \geq 0$
 $\Rightarrow (-\infty, 1]$ ✓

$y = \sqrt{\frac{x-1}{f(x)}} \rightarrow \frac{x-1}{f(x)} \geq 0$
 $\frac{-1}{-} \quad \frac{1}{+} \quad \frac{2}{-} \quad \frac{3}{+}$
 $D_f = (-1, 1] \cup (2, 3)$ ✓

$y_1 = \frac{1}{1x^2 - 1x^2 - 1x - 2} = \frac{1}{(1x^2 - 1x^2 + 1) - (x + 1x + 2)} = \frac{1}{(1x^2 - 1) - (x+2)} = \frac{1}{(1x^2 + x + 1)(1x^2 - x - 2)}$
 $\Rightarrow D_{y_1} = \mathbb{R} - \{1x^2 - x - 2 \text{ رضای}\} = D_{y_2} = \mathbb{R} - \{1x^2 + ax + b \text{ رضای}\}$
 $\Rightarrow 1x^2 - x - 2 = 1x^2 + ax + b \Rightarrow \begin{cases} a = -1 \\ b = -2 \end{cases} \Rightarrow a + b = -3 \quad -(a+b) = 3 \quad \sqrt{3} = 2$ ✓

$f(x) = x^x \cdot x$
 $f\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^{\frac{1}{x}} \cdot \frac{1}{x} = \frac{1}{x^{\frac{1}{x}}} \cdot \frac{1}{x} = \frac{1}{x^{\frac{1}{x} + 1}}$
 $y = \sqrt{f(x) - f\left(\frac{1}{x}\right)} \Rightarrow f(x) - f\left(\frac{1}{x}\right) \geq 0$
 $\Rightarrow \left(x^x - \frac{1}{x^{\frac{1}{x} + 1}}\right) + \left(x - \frac{1}{x}\right) \geq 0$
 $\Rightarrow \left(x - \frac{1}{x}\right) \left(x^x + \frac{1}{x^{\frac{1}{x} + 1}}\right) + \left(x - \frac{1}{x}\right) \geq 0 \Rightarrow \left(x - \frac{1}{x}\right) \left(x^x + \frac{1}{x^{\frac{1}{x} + 1}} + 1\right) \geq 0$
 $D_f = \mathbb{R} - (-2, 2)$

$x = \frac{1}{x}$
 $x^x = \frac{1}{x}$
 $x^x = \frac{1}{x} \Rightarrow x = \pm 2$
 $\frac{-2}{-} \quad \frac{0}{+} \quad \frac{2}{+}$
 $D = [-2, 0) \cup [2, +\infty)$

$$f(x^r, x) = 2x^r - 1$$

$$\xrightarrow{x=1} f(1, 1) = 2(1)^r - 1 \Rightarrow f(r) = 1$$

$$\xrightarrow{x=r} f(1, r) = 2(r)^r - 1 \Rightarrow f(1, r) = r$$

$$\Rightarrow f(r) + f(1, r) = 1 + r = 1 \quad \checkmark$$

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$$f(x) = x^r - 4x^r + 12x = x^r - 4x^r + 12x - 1 + 1 \Rightarrow (x-r)^r + 1 \Rightarrow f(\sqrt{r}, r) = (\sqrt{r})^r + 1 = 10$$

$$g(x) = x^r + 9x^r + 2rx = x^r + 9x^r + 2rx - 2r + 2r = (x-r)^r - 2r \Rightarrow g(\sqrt{r}, r) = (\sqrt{r})^r - 2r = -2$$

$$\frac{y_0}{10} = -2 \quad \checkmark$$

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$$f(x) = \sqrt{x \cdot \varepsilon \sqrt{x \cdot \varepsilon}} = \sqrt{x \cdot \varepsilon + \varepsilon \sqrt{x \cdot \varepsilon}} = \sqrt{(\sqrt{x \cdot \varepsilon} + \varepsilon)^2} = |\sqrt{x \cdot \varepsilon} + \varepsilon| = \sqrt{x \cdot \varepsilon} + \varepsilon$$

$$g(x) = \sqrt{x \cdot \varepsilon \sqrt{x \cdot \varepsilon}} = \sqrt{x \cdot \varepsilon - \varepsilon \sqrt{x \cdot \varepsilon}} = \sqrt{(\sqrt{x \cdot \varepsilon} - \varepsilon)^2} = |\sqrt{x \cdot \varepsilon} - \varepsilon| \Rightarrow \begin{cases} \sqrt{x \cdot \varepsilon} - \varepsilon = x \wedge \\ -\sqrt{x \cdot \varepsilon} + \varepsilon = \varepsilon \end{cases}$$

$$\Rightarrow f(x) + g(x) = \begin{cases} a & b \\ 0 + \sqrt{x \cdot \varepsilon} & \varepsilon + 0 \end{cases} \quad x \geq 1 \Rightarrow \frac{a+b}{c} = \frac{r}{-r} = -\frac{1}{r} \quad \checkmark$$

$$\text{منه نتوان این حالت را معلوم کرد} \quad \{x < 1 \Rightarrow \frac{a+b}{c} = \frac{r}{-r} = -1\}$$

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$$g(x) = \left\{ (1, 1), (1, 0), (2, 5), (0, \varepsilon) \right\} \Rightarrow Dg = \{1, 2, 0, r\}$$

$$f(x) = \frac{x+r}{x^r - \varepsilon x + r} \Rightarrow Df = \mathbb{R} - \{1, 2\}$$

$$f(r) = -\varepsilon$$

$$f(0) = \frac{r}{r}$$

$$\frac{g}{f} = \left\{ (2, -\frac{1}{\varepsilon}), (0, r) \right\} \quad \checkmark$$

$$\text{الف) } f(rx) = \left\{ (\frac{1}{r}, r), (\frac{r}{r}, -1), (r, r), (-\frac{1}{r}, r) \right\} \quad \checkmark$$

$$\text{ب) } f(x^r) = \left\{ (1, r), (-1, r), (\sqrt{r}, -1), (-\sqrt{r}, -1), (r, r), (-r, r) \right\} \quad \checkmark$$

$$\text{ج) } r g'(x) + 1 = \left\{ (-2, 19), (1, 3), (2, 9), (-1, 1) \right\} \quad \checkmark$$

$$\text{د) } \frac{rf}{g} = \left\{ (1, -\varepsilon), (2, -1) \right\} \quad \checkmark$$

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