

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \Rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \Rightarrow \frac{x^2 - 2x + 1 - x^2}{x(x-1)} \geq 0 \Rightarrow \frac{1-2x}{x(x-1)} \geq 0$ (۲)

$\Rightarrow$ $\frac{0}{+} \frac{1}{-} \frac{1}{+} \frac{1}{-}$ $\Rightarrow D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$ ✓

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{1}{x}}{\frac{1}{x-1} + \frac{1}{x+2}} \Rightarrow \frac{x \neq 0, -1, -2}{\frac{1}{x-1} + \frac{1}{x+2} = 0} \Rightarrow \frac{1}{x-1} = -\frac{1}{x+2} \Rightarrow 1+x = -x-2 \Rightarrow x = -\frac{3}{2}$

$\Rightarrow x = -\frac{3}{2} \Rightarrow x = -\frac{3}{2} = -1.5$ $D_f = \mathbb{R} - \{-\frac{3}{2}, -1, 0, -2, 1\}$ ✓

الف) $f(x) = \sqrt{((\frac{1}{x})^2 - 9)(3x - \varepsilon^2)} \Rightarrow ((\frac{1}{x})^2 - 9)(3x - \varepsilon^2) \geq 0$ $D_f = \mathbb{R} - (-2, 2)$ ✓

$(\frac{1}{x})^2 - 9 = 0 \Rightarrow (\frac{1}{x})^2 = 9 \Rightarrow \frac{1}{x^2} = 9 \Rightarrow x^2 = \frac{1}{9} \Rightarrow x = \pm \frac{1}{3}$

$3x - \varepsilon^2 = 0 \Rightarrow 3x = \varepsilon^2 \Rightarrow x = \frac{\varepsilon^2}{3}$

ب) $\sqrt{x+1} + \sqrt{y+1} = 3 \Rightarrow \sqrt{y+1} = 3 - \sqrt{x+1} \Rightarrow y+1 = (3 - \sqrt{x+1})^2$

$\Rightarrow y = (3 - \sqrt{x+1})^2 - 1 \Rightarrow x \geq -1$ $D_f = [1, +\infty) \cap (-\infty, 10] = [1, 10]$ ✓

$f(x) = \frac{\log(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$ (۲)

(I) $x^2 - x - 2 > 0 \Rightarrow x^2 - x - 2 > 0 \Rightarrow (x-2)(x+1) > 0 \Rightarrow \frac{-1}{+} \frac{2}{-} \frac{2}{+} \frac{1}{-} \Rightarrow (-\infty, -1) \cup (2, +\infty)$ ۳

(II) $x^2 - 1 > 0 \Rightarrow x = \pm 1 \Rightarrow \frac{-1}{+} \frac{1}{-} \frac{1}{+} \frac{1}{-} \Rightarrow (-\infty, -1] \cup [1, +\infty)$

(III) $\sqrt{x^2 - 1} + 1 = 0 \Rightarrow \sqrt{x^2 - 1} = -1 \times$

$D_f = I \cap II \Rightarrow (-\infty, -1) \cup (2, +\infty)$ ✓

$\sqrt{3+ax-x^2}$ $D_f = [-2, b] \Rightarrow \frac{-2}{-} \frac{b}{+} \frac{b}{-}$ (۲)

$3+ax-x^2 \xrightarrow{x=-2} 3-2a-\varepsilon=0 \Rightarrow -2a=1 \Rightarrow a=-\frac{1}{2}$

$\Rightarrow \sqrt{3-\frac{1}{2}x-x^2} \Rightarrow -x^2 - \frac{1}{2}x + 3 \geq 0 \Rightarrow -x^2 - \frac{1}{2}x + 3 = 0 \Rightarrow \Delta = \frac{1}{4} + 12 \Rightarrow \Delta = \frac{49}{4}$ ۴

$\Rightarrow x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{\frac{1}{2} \pm \frac{7}{2}}{-2} \Rightarrow \frac{\frac{1}{2} + \frac{7}{2}}{-2} = -2$ $a = -\frac{1}{2}$

$\frac{\frac{1}{2} - \frac{7}{2}}{-2} = 3$ $b = \frac{3}{2}$

$a+b = -\frac{1}{2} + \frac{3}{2} = 1$ ✓

$f(x) = \begin{cases} 3x-2 & ; x \geq 1 \\ 2x+3 & ; x < 1 \end{cases}$ $g(x) = \sqrt{f(x)-x}$ (۲)

if $x \geq 1 \Rightarrow g(x) = \sqrt{3x-2-x} = \sqrt{2x-2} \Rightarrow 2x-2 \geq 0 \Rightarrow 2x \geq 2 \Rightarrow x \geq 1$ ✓

if $x < 1 \Rightarrow g(x) = \sqrt{2x+3-x} = \sqrt{x+3} \Rightarrow x+3 \geq 0 \Rightarrow x \geq -3 \Rightarrow -3 \leq x < 1$ ✓

$D_f = [-3, 1) \cup [1, +\infty) = [-3, +\infty)$ ✓

$$f(x) = \begin{cases} (a+1)(x+2) & ; x > 1 \\ 3a+2x & ; x \leq 1 \end{cases} \quad 2f(0) = f(-2) + a \quad (I)$$

$$2f(0) = 2 \times (a+1)(0+2) = 2 \times (a+1) \times 2 = 4a+4$$

$$f(-2) = 3a+2(-2) = 3a-4$$

$$(I) = 4a+4 = 3a-4+a \Rightarrow 4a+4 = 4a-4 \Rightarrow 10a = -8 \Rightarrow a = -\frac{4}{5}$$

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2 \quad f(\sqrt{2}-\sqrt{3}) + f(\sqrt{2}+\sqrt{3}) = 8 = 2A = \sqrt{2} + 2\sqrt{6} \quad \checkmark$$

$$f(\sqrt{2}-\sqrt{3}) + f(\sqrt{2}+\sqrt{3}) = \sqrt{\sqrt{2}-\sqrt{3}} + \frac{1}{\sqrt{\sqrt{2}-\sqrt{3}}} + 2 + \sqrt{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{\sqrt{2}+\sqrt{3}}} + 2$$

$$\Rightarrow f(\sqrt{2}-\sqrt{3}) + f(\sqrt{2}+\sqrt{3}) = \frac{2}{\sqrt{\sqrt{2}+\sqrt{3}}} + \frac{2}{\sqrt{\sqrt{2}-\sqrt{3}}} + 4 = \frac{2\sqrt{\sqrt{2}-\sqrt{3}} + 2\sqrt{\sqrt{2}+\sqrt{3}}}{(\sqrt{\sqrt{2}+\sqrt{3}})(\sqrt{\sqrt{2}-\sqrt{3}})} + 4$$

$$= 2\sqrt{\sqrt{2}-\sqrt{3}} + 2\sqrt{\sqrt{2}+\sqrt{3}} + 4 = 2A \Rightarrow A = \sqrt{\sqrt{2}-\sqrt{3}} + \sqrt{\sqrt{2}+\sqrt{3}} + 2 \Rightarrow A - 2 = \sqrt{\sqrt{2}-\sqrt{3}} + \sqrt{\sqrt{2}+\sqrt{3}}$$

$$\xrightarrow{\text{مربع}} A^2 - 4A + 4 = \sqrt{2}-\sqrt{3} + \sqrt{2}+\sqrt{3} + 2(\sqrt{\sqrt{2}-\sqrt{3}}\sqrt{\sqrt{2}+\sqrt{3}}) \Rightarrow A^2 - 4A - 2 = 0 \Rightarrow A = \frac{4 \pm \sqrt{16+8}}{2} = \frac{4 \pm \sqrt{24}}{2} = \frac{4 \pm 2\sqrt{6}}{2} = 2 \pm \sqrt{6}$$

$$\begin{cases} 2f(x) - 3f(-x) = \varepsilon x^2 - x \\ 2f(-x) - 3f(x) = \varepsilon x^2 + x \end{cases} \Rightarrow \omega f(-x) - \omega f(x) = 2x \Rightarrow f(-x) - f(x) = \frac{2}{\omega}x$$

$$2f(-x) - 3f(x) = \varepsilon x^2 + x \Rightarrow 2f(-x) - 2f(x) - f(x) = \varepsilon x^2 + x$$

$$\Rightarrow \frac{\varepsilon}{\omega}x - f(x) = \varepsilon x^2 + x \Rightarrow -f(x) = \varepsilon x^2 + \frac{x}{\omega} \Rightarrow f(x) = -\varepsilon x^2 - \frac{x}{\omega} \quad \checkmark$$

$$(x+2)f(x) - 3xf(x+2) = \varepsilon x^2 - mx + 3m-1 \quad f(0) = 9$$

$$\text{if } x=0 \Rightarrow 2f(0) = 3m-1 \quad (I)$$

$$\text{if } x=-2 \Rightarrow 4f(0) = 19+2m+3m-1 \Rightarrow 4f(0) = 5m+18 \Rightarrow 2f(0) = \frac{5m+18}{2} \quad (II)$$

$$I, II \Rightarrow 3m-1 = \frac{5m+18}{2} \Rightarrow 6m-2 = 5m+18 \Rightarrow \varepsilon m = 20 = m = \frac{9}{2} \quad \checkmark$$

$$2f(0) = \frac{5 \times 9}{2} - 1 = 13, 18 - 1 = 17, 18 \Rightarrow f(0) = \frac{17, 18}{2} = \frac{17}{2} \quad \checkmark$$

$$f(x) + f\left(\frac{1}{x}\right) = \frac{2x^2 - 12x + 3}{x} \quad f(-1) = 9$$

$$\text{if } x=-1 \Rightarrow f(-1) + f(-1) = \frac{2+12+3}{-1} \Rightarrow 2f(-1) = -17 \Rightarrow f(-1) = -\frac{17}{2} \quad \checkmark$$

$$\text{if } x=1 \Rightarrow f(1) + f(1) = \frac{2-12+3}{1} = -7 \Rightarrow 2f(1) = -7 \Rightarrow f(1) = -\frac{7}{2}$$

$$\text{شیب} = a = \frac{3 - (-7)}{1 - (-1)} = \frac{10}{2} = 5 \quad f(x) = 5x + b \xrightarrow{f(1)=-\frac{7}{2}} 5+b = -\frac{7}{2} \Rightarrow b = -\frac{17}{2}$$