

الف)  $f(x) = \sqrt{\frac{x-1}{x}} - \frac{x}{x-1}$  (۲)

ب)  $f(x) = \frac{1}{x+1} - \frac{2}{x}$  (۲)

$\frac{1}{x+1} - \frac{2}{x} = \frac{x - 2(x+1)}{x(x+1)} = \frac{-x-2}{x^2+x}$

$\frac{-x-2}{x^2+x} \neq 0 \Rightarrow x^2+x \neq 0 \Rightarrow x(x+1) \neq 0 \Rightarrow x \neq 0, -1$

$D_f = (-\infty, 0) \cup (0, -1) \cup (-1, +\infty)$

الف)  $f(x) = \sqrt{\left(\left(\frac{1}{3}\right)^x - 9\right)(32 - 4^x)}$  (۲)

ب)  $f(x) = 4\sqrt{x-1} + \sqrt{y+1} = 3$

$y+1 = 3 - 4\sqrt{x-1}$

$y = 8 + \sqrt{x-1} - 4\sqrt{x-1}$

$y = 8 - 3\sqrt{x-1}$

$y+1 \geq 0 \Rightarrow 8 - 3\sqrt{x-1} + 1 \geq 0 \Rightarrow 9 \geq 3\sqrt{x-1} \Rightarrow 3 \geq \sqrt{x-1} \Rightarrow 9 \geq x-1 \Rightarrow x \leq 10$

$x-1 \geq 0 \Rightarrow x \geq 1$

$D_f = [1, 10]$

$f(x) = \frac{\log_2(x^2-x-2)}{\sqrt{x^2-1}+1}$  (۲)

$x^2-x-2 > 0 \Rightarrow (x-2)(x+1) > 0$

$D_f = (-\infty, -1) \cup (2, +\infty)$

$x^2-1 \geq 0 \Rightarrow x^2 \geq 1 \Rightarrow x \geq 1 \vee x \leq -1$

$x \geq 1 \vee x \leq -1$

$-2, b$  ریشه های  $\sqrt{3+ax-x^2}$  هستند پس (۲)

$3-2a-4=0 \Rightarrow a=0/5 \rightarrow \sqrt{3-0/5x-x^2} \rightarrow \Delta = 0/25 + 12 \cdot 12/25$

$\frac{-b \pm \sqrt{\Delta}}{2a} \Rightarrow \frac{0/5 \pm 3/5}{-2} = \frac{-3}{4} \rightarrow \frac{3}{4} \rightarrow \frac{1}{5}, b$

$D_f = [1/5, b]$

$f(x) \geq x \rightarrow f(x) \geq x$  (۲)

$3x-2 \geq x \rightarrow 2x \geq 2 \Rightarrow x \geq 1$

$2x+3 \geq x \rightarrow x \geq -3$

$x \geq -3 \rightarrow -3 \leq x < 1$

$D_f = [-3, +\infty)$

$$f(x) = \begin{cases} (a+1)(x+2) & ; x > 1 \\ 3a+2x & ; x \leq 1 \end{cases} \quad 2f(5) - f(-2) + a$$

بابتی

$$2(7a+7) = 3a-4+a \rightarrow 14a+14 = 4a-4-10a-18$$

$$a = -1/8 \quad \checkmark$$

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2 \quad f(2-\sqrt{3}) + f(2+\sqrt{3})$$

$$\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2-\sqrt{3}}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 4$$

$$\rightarrow \sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}} + \frac{\sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}}}{\sqrt{4-3}} + 4$$

$$2\sqrt{2-\sqrt{3}} + 2\sqrt{2+\sqrt{3}} + 4 = \sqrt{8-2\sqrt{12}} + \sqrt{8+2\sqrt{12}} + 4$$

$$= \sqrt{6-\sqrt{2}} + \sqrt{6+\sqrt{2}} + 4$$

$$2\sqrt{6+4} \quad \checkmark$$

$$2 \times 2f(x) - 3f(-x) = 4x^2 - x$$

$$3 \times 2f(-x) - 3f(x) = 4x^2 + x$$

$$= -5f(x) - 20x^2 + x$$

$$f(x) = -4x^2 - \frac{x}{5} \quad \checkmark$$

$$(x+2)f(x) - 3xf(x+2) = 4x^2 - mx + 3m - 1$$

$$x=0 \rightarrow 2f(0) = 3m-1 \rightarrow f(0) = 1/5m - 0/5 \quad 6/75$$

$$x=2 \rightarrow 6f(0) = 16+2m+3m-1 \rightarrow f(0) = 1/5(4/5) - 0/5 = 6/25 \quad \checkmark$$

$$16+5m-1 = 9m-3 \rightarrow 16-1+3=9m-m \rightarrow m = \frac{18}{4} = \frac{9}{2} = 4/5$$

$$f(x) + f\left(\frac{1}{x}\right) = \frac{3x^2 - 12x + 3}{x}$$

$$x=-1 \rightarrow f(-1) + f(-1) = \frac{3+12+3}{-1} = -18 = 2f(-1)$$

$$f(-1) = -9 \quad \checkmark$$