

الف) $x \neq 0$ $\frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{x^2-2x+1-x^2}{x^2-x} \geq 0$
 $\rightarrow \frac{-2x+1}{x^2-x} \geq 0$ $\frac{0}{+ \quad -} \frac{1}{+ \quad -}$ $D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$ ✓ (۲)

ب) $x+1 \neq 0 \rightarrow x \neq -1$ $x \neq 0$ $x-1 \neq 0 \rightarrow x \neq 1$ $D_f = \mathbb{R} - \{-1, -\frac{1}{2}, 1\}$
 $x+2 \neq 0 \rightarrow x \neq -2$ $\frac{3}{x+1} + \frac{1}{x+1} \neq 0 \rightarrow \frac{4x+4}{(x+1)(x-1)} \neq 0$ $x \neq -\frac{4}{4} = -1$ ✓

$(x^2-3^2)(x^2-2^2) \geq 0$ $\frac{-2}{+ \quad -} \frac{2, \infty}{- \quad +}$
 $D_f = (-\infty, -2] \cup [2, \infty, +\infty)$ ✓ (۲)

ب) $x-1 \geq 0 \rightarrow x \geq 1$ $3 - \sqrt{x-1} \geq 0 \rightarrow 3 \geq \sqrt{x-1}$
 $\xrightarrow{\text{کوبه}} 1 \geq x-1 \Rightarrow 1^2 \geq x$ $D_f = [1, 1^2]$ ✓

$x^2-x-2 \geq 0 \rightarrow (x-2)(x+1) \geq 0$ $\frac{-1}{+ \quad -} \frac{2}{- \quad +} \rightarrow \bar{I}$
 $x^2-1 \geq 0 \rightarrow (x-1)(x+1) \geq 0$ $\frac{-1}{+ \quad -} \frac{1}{- \quad +} \rightarrow \bar{I}$
 $\sqrt{x^2-1} \neq -1 \rightarrow$ معادله برقرار
 $\Rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$ ✓ (۲)

$x = -\frac{2}{f} \rightarrow -\frac{4}{f} - 2x + 3 = 0 \Rightarrow a = -\frac{1}{f}$ $-x^2 - \frac{1}{f}x + 3 = 0$
 $\Rightarrow -(x + \frac{1}{f})(x - \frac{3}{f}) = 0 \Rightarrow b = \frac{3}{f}$ $a+b = 1$ ✓ (۲)

$\begin{cases} 3x-2 \geq x \rightarrow 2x-2 \geq 0 \rightarrow x \geq 1 \\ 2x+3 \geq x \rightarrow x+3 \geq 0 \rightarrow x \geq -3 \rightarrow -3 \leq x < 1 \end{cases}$
 $\Rightarrow D_g = [-3, +\infty)$ ✓ (۲)

$$2(\sqrt{a} + \sqrt{b}) = 2a - 1 + a \rightarrow 1\sqrt{a} + 1\sqrt{b} = 2a - 1 \rightarrow 10a = -11$$

$$\rightarrow \underline{a = -1, 1} \quad \checkmark$$

4
6

$$(\sqrt{x} - \sqrt{y})(\sqrt{x} + \sqrt{y}) = 1 \quad \sqrt{x} - \sqrt{y} = b \quad \sqrt{x} + \sqrt{y} = a$$

$$t = \sqrt{a} + \frac{1}{\sqrt{a}} \quad f(a) + f(b) = (\sqrt{a} + \frac{1}{\sqrt{a}} + \sqrt{y}) + (\sqrt{b} + \frac{1}{\sqrt{b}} + \sqrt{y})$$

2
7

$$\sqrt{b} + \frac{1}{\sqrt{b}} = \sqrt{a} + \frac{1}{\sqrt{a}} \Rightarrow \sqrt{y} + \frac{1}{\sqrt{y}} = \sqrt{a} + \frac{1}{\sqrt{a}} \Rightarrow \sqrt{y} = \sqrt{a} + \frac{1}{\sqrt{a}} + \sqrt{y} \quad a + \frac{1}{a} = \sqrt{y} + \sqrt{y} + (\sqrt{y} - \sqrt{y}) = \sqrt{y}$$

$$f(y) = \sqrt{y} + \frac{1}{\sqrt{y}} = \sqrt{y} \Rightarrow t = \sqrt{y} \quad f(a) + f(b) = \sqrt{y} + \sqrt{y} + \sqrt{y} = 3\sqrt{y} + \sqrt{y} \quad \checkmark$$

$$\left. \begin{aligned} 2f(a) - 3f(-a) &= 2a^2 - a \\ 2f(-a) - 3f(a) &= 2a^2 + a \end{aligned} \right\} \Rightarrow -a f(a) = 2a^2 + a$$

2
8

$$\Rightarrow \underline{f(a) = -2a - \frac{1}{a}} \quad \checkmark$$

$$\begin{aligned} x=1 &\rightarrow f(0) = 0m + 1d \rightarrow 1 \wedge f(0) = 1d m + 1d \\ x=0 &\rightarrow 2f(0) = 2m - 1 \rightarrow 10f(0) = 1d m - 1d \end{aligned} \Rightarrow \boxed{f(0) = \frac{1d}{1}} \quad \checkmark$$

2
9

$$\frac{x(2n-1) + 2}{n} = 2n - 1 + \frac{2}{n} = 2n - 4 + \frac{2}{n} - 4$$

$$f(x) + f\left(\frac{1}{x}\right) = 2n - 4 + \frac{2}{n} - 4 \Rightarrow f(x) = 2n - 4$$

$$f\left(\frac{1}{x}\right) = \frac{2}{n} - 4$$

$$\underline{f(-1) = -3 - 4 = -7} \quad \checkmark$$

2
10