

ب: دهم سر

تکلیف

امروزه بکسر و سر

الف) $f(x) = \sqrt{-\frac{x-1}{x} - \frac{x}{x-1}} \rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0$

$\rightarrow \frac{x^2 - (x+1) - x^2}{x(x-1)} \geq 0 \rightarrow \frac{-x-1}{x(x-1)} \leq 0$

$\rightarrow \frac{0}{-1} \frac{1}{0} \frac{1}{-1} \rightarrow D_f = (-\infty, 0) \cup (1, +\infty)$

ب)

$f(x) = \frac{\frac{1}{x+1} - \frac{1}{x}}{\frac{1}{x-1} + \frac{1}{x+2}} = \frac{\frac{x - (x+1)}{x(x+1)}}{\frac{x+2 + x-1}{(x-1)(x+2)}} = \frac{-1}{\frac{(x-1)(x+2)}{x(x+1)}}$

$\rightarrow D_f = \mathbb{R} - \{1, 0, -1, -2, -\frac{1}{2}\}$

الف) $f(x) = \sqrt{\left(\left(\frac{1}{x}\right)^x - 9\right)(3x - 5^x)} \rightarrow \left(\left(\frac{1}{x}\right)^x - 9\right)(3x - 5^x) \geq 0$

$\rightarrow \frac{-2}{+1} \frac{0}{-1} \rightarrow D_f = (-\infty, -2] \cup \left[\frac{1}{2}, +\infty\right)$

ب) $\sqrt{x-1} + \sqrt{y+1} = 3 \rightarrow \sqrt{y+1} = 3 - \sqrt{x-1}$

$\rightarrow \begin{cases} \text{I) } x-1 \geq 0 \rightarrow x \geq 1 \\ \text{II) } \sqrt{x-1} \leq 3 \rightarrow x-1 \leq 9 \rightarrow x \leq 10 \end{cases} \rightarrow D_f = [1, 10]$

ج) $f(x) = \frac{\log(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \rightarrow \begin{cases} \text{I) } x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0 \rightarrow \frac{-1}{+1} \frac{2}{-2} \rightarrow (-\infty, -1) \cup (2, +\infty) \\ \text{II) } x^2 - 1 \geq 0 \rightarrow x^2 \geq 1 \rightarrow x \geq 1 \text{ or } x \leq -1 \rightarrow (-\infty, -1] \cup [1, +\infty) \\ \text{III) } \sqrt{x^2 - 1} + 1 \neq 0 \rightarrow \sqrt{x^2 - 1} \neq -1 \rightarrow \mathbb{R} \end{cases}$

$\text{I) II) III) } D_f = (-\infty, -1) \cup (2, +\infty)$

د) $\sqrt{3+ax-n^2} \rightarrow D_f = [-2, b]$

$\rightarrow -n^2 + ax + 3 \geq 0 \xrightarrow{a \neq 0} \frac{a}{b^2 - 3} > 0$
 $\rightarrow x \geq -2 \rightarrow -3 - 2a + 3 = 0 \rightarrow -2a = 0 \rightarrow a = 0$
 $\rightarrow x \leq b \rightarrow -b^2 + ab + 3 = 0 \rightarrow -b^2 - \frac{b}{2} + 3 = 0$
 $\rightarrow b = \frac{1}{2} \rightarrow a + b = -\frac{1}{2} + \frac{1}{2} = 0$

ω) $g(x) = \sqrt{f(x)} - x \rightarrow f(x) - x \geq 0$

$f(x) = \begin{cases} 3x-2, & x \geq 1 \\ x+3, & x < 1 \end{cases} \rightarrow f(x) - x = \begin{cases} 3x-2-x, & x \geq 1 \rightarrow x \geq 1 \\ x+3-x, & x < 1 \rightarrow x \geq -3 \end{cases}$
 $\Rightarrow \mathbb{R} \rightarrow Df = [-3, +\infty)$

ξ) $f(x) = \begin{cases} (a+1)(x-2) & x > 1 \\ 3a+2x & x \leq 1 \end{cases} \quad f(0) = f(-2) + a$

$\rightarrow f(0) = f(-2) + a \rightarrow (a+1)(-2) = 3a + (-2) + a$
 $\rightarrow -2a - 2 = 4a - 2 \rightarrow -2a - 2 = 4a - 2 \rightarrow -6a = 0 \rightarrow a = 0$

ν) $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2 \quad f(\sqrt{x}+2) + f(2-\sqrt{x})$

$\Downarrow$
 $f(x) = \sqrt{x + \frac{1}{x}} + 2 + 2 \quad (\sqrt{x}+2)(2-\sqrt{x}) = 1 \rightarrow$
 $\rightarrow f(2-\sqrt{x}) + f(\sqrt{x}+2) = 2f(\sqrt{x}+2)$
 $\rightarrow f(\sqrt{x}+2) = \sqrt{2+\sqrt{x} + \frac{1}{2+\sqrt{x}}} + 2 + 2 = \sqrt{2+\sqrt{x} + \frac{1}{2+\sqrt{x}}} + 4 = \sqrt{4+2\sqrt{x}} + 4$
 $\rightarrow 2f(\sqrt{x}+2) = 2(\sqrt{4+2\sqrt{x}} + 4)$

η) $\begin{cases} f(x) - 3f(-x) = 2x^2 - x \\ 4f(-x) - 9f(x) = 12x^2 + 3x \end{cases} \rightarrow \begin{cases} f(x) - 3f(-x) = 2x^2 - x \\ 4f(-x) - 9f(x) = 12x^2 + 3x \end{cases}$
 $\rightarrow f(x) = 2x^2 - \frac{1}{5}x$

θ) $(a+2)f(a) - 3af(a+2) = 2a^2 - ma + 3m-1$

$\begin{matrix} x=2 \\ x=0 \end{matrix} \rightarrow \begin{cases} 4f(0) = 12 + 2m + 3m-1 \\ 2f(0) = 3m-1 \end{cases} \Rightarrow \begin{cases} 2m+10 = 9m-3 \\ \sum m = 14 \\ m = 14/7 = 2 \end{cases}$
 $\rightarrow f(0) = 2(2) = 4$

ι) $x=1 \rightarrow f(-1) + f(1) = \frac{3+1^2 + 1^2}{-1} = -2$
 $\rightarrow 2f(-1) = -2 \rightarrow f(-1) = -1$