۱)

$x \neq 0$

$x-1 \neq 0 \Rightarrow x \neq 1$

$f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}}$

$\frac{(x-1)^2-x^2}{x(x-1)}=\frac{\cancel{x^2}+1-2x-\cancel{x^2}}{x^2-x} \rightarrow \frac{1}{2}$

$\begin{array}{ccccccc} & 0 & & \frac{1}{2} & & 1 & \\ + & | & - & | & + & | & - \end{array}$

$\frac{1}{2} \quad 1$

$D_f=(-\infty \quad 0)\cup[\frac{1}{2} \quad 1)$

$f(x)=\dfrac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}}$

$x \neq -1 , 0 , 1 , -2$

$x \neq -\frac{5}{4}$

$4x+5$

$\frac{3}{x-1}+\frac{1}{x+2} \neq 0$

$\frac{3x+6+x-1}{(x-1)(x+2)}$

$D_f=R-\{-1 , 0 , 1 , -2 , -\frac{5}{4}\}$

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۲)

$f(x)=\sqrt{\underbrace{((\frac{1}{3})^x-9)}_{x=-2}\underbrace{(32-4^x)}_{x=\frac{5}{2}}}$

$\begin{array}{ccccc} & -2 & & \frac{5}{2} & \\ + & \phi & - & \phi & + \end{array}$

$D_f=(-\infty \quad -2]\cup[\frac{5}{2} \quad +\infty)$

$\sqrt[4]{x-1}+\sqrt{y+1}=3$

$x-1 \geq 0 \quad x \geq 1$

$\sqrt{y+1}=3-\sqrt[4]{x-1} \Rightarrow 3-\sqrt[4]{x-1} \geq 0$

$x-1 \leq 81$

$x \leq 82$

$D_f=[1 \quad 82]$

$f(x)=\dfrac{\log_2(x^2-x-2)}{\sqrt{x^2-1}+1}$

$x^2-x-2>0$

$\frac{-1}{2}$

$\begin{array}{ccccc} & -1 & & 2 & \\ + & | & - & | & + \end{array}$

$\sqrt{x^2-1}+1 \neq 0$

هیچگاه ۰ نمی‌شود

$x^2-1 \geq 0$

$\begin{array}{ccccc} & -1 & & 1 & \\ + & | & - & | & + \end{array}$

$D_f=(-\infty \quad -1)\cup(2 \quad +\infty)$

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۴-

$\sqrt{3+ax-x^2}$

$-(x+2)(x-\frac{3}{2}) = -x^2 - \frac{1}{2}x + 3 \Rightarrow a = -\frac{1}{2}$

$b = \frac{3}{2}$

$\frac{3}{2} + (-\frac{1}{2}) = 1$

۵-

$f(x) = \begin{cases} 3x-2 & ; x \geq 1 \\ 2x+3 & ; x < 1 \end{cases}$

$f(x) - x \geq 0 \xrightarrow{x \geq 1} 3x - 2 - x = 2x - 2 \Rightarrow x \geq 1$

$f(x) - x \geq 0 \xrightarrow{x<1} 2x + 3 - x = x + 3 \Rightarrow x \geq -3$

$D_f = [-3, +\infty)$

۶-

$2f(a) = f(-2) + a \qquad f(x) = \begin{cases} (x+1)(x+2) & ; x \geq 1 \\ 3x + 2x & ; x < 1 \end{cases}$

$|2a+1|^2 = 3a - 4 + a$

$10a = -18$

$a = -1.8$

۷-

$f(2-\sqrt{3}) + f(2+\sqrt{3})$

$\sqrt{2-\sqrt{3}} + \frac{1}{\sqrt{2-\sqrt{3}}} + 2 + \sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 2 = \cdot \quad \times\frac{\sqrt{2}}{\sqrt{2}}$

$\frac{\sqrt{4-2\sqrt{3}}}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{4-2\sqrt{3}}} + \frac{\sqrt{4+2\sqrt{3}}}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{4+2\sqrt{3}}} + 4 =$

$\frac{\sqrt{3}-1}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{3}-1} + \frac{\sqrt{3}+1}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{3}+1} + 4 \Rightarrow \frac{2\sqrt{3}}{\sqrt{2}} + \frac{\cancel{\sqrt{2}}(\sqrt{3}-1)}{\cancel{2}\sqrt{2}} + \frac{\cancel{2}(\sqrt{3}+1)}{2\sqrt{2}} + 4$

$$\frac{4\sqrt{3}}{\sqrt{2}} + 4 = \cancel{2\sqrt{6}+4} \quad 2\sqrt{6} + 4$$

-8

$$2f(x) - 3f(-x) = 4x^2 - x$$

$\xrightarrow{x=-x}$ $$2f(-x) - 3f(x) = 4x^2 + x$$

$$4f(x) - 6f(-x) = 8x^2 - 2x$$

$$6f(-x) - 9f(x) = 12x^2 + 3x$$

$$-5f(x) = 20x^2 + x$$

$$f(x) = \frac{20x^2 + x}{-5} = -4x^2 - \frac{x}{5}$$

-9

$$(x+2)f(x) - 3x f(x+2) = 4x^2 - mx + 3m - 1$$

$\xrightarrow{f(0)}$ $2(f(0)) - 0 = 3m - 1$ $\quad 2f(0) = 3m - 1 \quad \xrightarrow{\times 3} \quad 6f(0) = 9m - 3$

$\xrightarrow{f(-2)}$ $0 + 6f(0) = 16 + 2m + 3m - 1 \quad 6f(0) = 5m + 15$

$$9m - 3 = 5m + 15$$

$$4m = 18$$

$$6.25 = f(0) = \frac{25}{4} \Leftarrow \quad m = 4.5$$

-10

$$f(x) + f\left(\frac{1}{x}\right) = \frac{3x^2 - 12x + 3}{x}$$

$$= ax + b + \frac{a}{x} + b = 3x + \frac{3}{x} - 12$$

$\cancel{2f(1) = -12}$ $\quad \cancel{f(1) = -6}$

$\cancel{2f(1) = -12}$

$\cancel{f(1) = -6}$

$a = 3$

$2b = -12$

$b = -6$

$$f(x) = 3x - 6 \quad \xrightarrow{f(-1)} \quad f(-1) = -3 - 6 = -9$$

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