

الف) $f(x) = \sqrt{\frac{x-1}{x}} \cdot \frac{x}{x-1} \rightarrow \frac{(x-1)(x-1) - (x)(x)}{x(x-1)} = \frac{x^2 - 2x - x^2}{x(x-1)} = \frac{-2x}{x(x-1)} = \frac{-2}{x-1}$

$D_f = (-\infty, 0) \cup \left[\frac{1}{2}, 1\right)$ ✓

ب) $f(x) = \frac{1}{x+1} - \frac{2}{x} = \frac{x - 2(x+1)}{x(x+1)} = \frac{x - 2x - 2}{x(x+1)} = \frac{-x-2}{x(x+1)}$

$\frac{-x-2}{x(x+1)} = 0 \Rightarrow -x-2=0 \Rightarrow x=-2$

$\frac{-x-2}{x(x+1)} > 0 \Rightarrow \frac{-x-2}{x(x+1)} > 0 \Rightarrow x < -2 \text{ or } -1 < x < 0$

$\frac{-x-2}{x(x+1)} < 0 \Rightarrow \frac{-x-2}{x(x+1)} < 0 \Rightarrow -2 < x < -1 \text{ or } x > 0$

$D_f = (-\infty, -2) \cup (-1, 0) \cup (0, +\infty)$ ✓

الف) $f(x) = \sqrt{\left(\frac{1}{x} - 9\right)(32 - x^2)} \Rightarrow \frac{1}{x} - 9 = 0 \Rightarrow x = -\frac{1}{9}$

$32 - x^2 = 0 \Rightarrow x^2 = 32 \Rightarrow x = \pm\sqrt{32} = \pm 4\sqrt{2}$

$D_f = (-\infty, -\frac{1}{9}] \cup [\frac{1}{9}, +\infty)$ ✓

ب) $\sqrt{x-1} + \sqrt{x+1} = 3 \Rightarrow x \geq 1 \text{ and } x \geq -1$

$\sqrt{x+1} = 3 - \sqrt{x-1} \Rightarrow 3 - \sqrt{x-1} \geq 0 \Rightarrow \sqrt{x-1} \leq 3 \Rightarrow x-1 \leq 9 \Rightarrow x \leq 10$

$D_f = [1, 10]$ ✓

$f(x) = \frac{\log_4(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \rightarrow x^2 - x - 2 > 0 \Rightarrow (x-2)(x+1) > 0$

$\sqrt{x^2 - 1} + 1 \neq 0 \Rightarrow x^2 - 1 \geq 0 \Rightarrow x \leq -1 \text{ or } x \geq 1$

$D_f = (-\infty, -1) \cup (2, +\infty)$ ✓

$\sqrt{3+4x-x^2} \rightarrow [-2, 3]$

$f(x) = \sqrt{3 - \frac{1}{x} - x^2} \Rightarrow 3 - \frac{1}{x} - x^2 \geq 0 \Rightarrow 3x - 1 - x^3 \geq 0$

$\Rightarrow x^3 + x - 3 \leq 0 \Rightarrow x^3 + x - 12 \leq 0 \Rightarrow (x+3)(x-2) \leq 0$

$a + b = -\frac{1}{2} + \frac{3}{2} = 1$ ✓

$D_f = [-2, 1] \cup [1, +\infty)$ ✓

$g(x) = \sqrt{f(x) - x}$

$f(x) = \begin{cases} 2x-2 & x \geq 1 \\ 2x+3 & x < 1 \end{cases}$

$D_f = [-3, 1) \cup [1, +\infty)$ ✓

$f(x) - x \geq 0 \Rightarrow \begin{cases} 2x-2-x \geq 0 & x \geq 1 \\ 2x+3-x \geq 0 & x < 1 \end{cases}$

$\Rightarrow \begin{cases} x-2 \geq 0 \Rightarrow x \geq 2 \\ x+3 \geq 0 \Rightarrow x \geq -3 \end{cases}$

$\Rightarrow -3 \leq x < 1$

$$f(a) = \begin{cases} (a+1)(a+1) & ; a > 1 \\ ra + r & ; a \leq 1 \end{cases} \Rightarrow f(a) = (a+1)(a+1) = V(a+1)$$

$$\Rightarrow f(a) = f(-r) + a \Rightarrow 1 + (a+1) = ra - r + a$$

$$\Rightarrow 1 + a + 1 = ra - r \Rightarrow 1 + a = ra - r \Rightarrow \boxed{a = -1, 1} \checkmark$$

$$f(a) = \sqrt{x} + \frac{1}{\sqrt{x}} + r$$

$$f(r-\sqrt{x}) + f(r+\sqrt{x}) = \sqrt{r-\sqrt{x}} + \frac{1}{\sqrt{r-\sqrt{x}}} + r + \sqrt{r+\sqrt{x}} + \frac{1}{\sqrt{r+\sqrt{x}}} + r =$$

$$\Rightarrow \frac{\sqrt{r-\sqrt{x}}}{\sqrt{x}} + \frac{\sqrt{x}}{\sqrt{r-\sqrt{x}}} + \frac{\sqrt{r+\sqrt{x}}}{\sqrt{x}} + \frac{\sqrt{x}}{\sqrt{r+\sqrt{x}}} = 2$$

$$\Rightarrow \frac{r\sqrt{x}}{\sqrt{x}} + \frac{\sqrt{x}(\sqrt{x}+1) + \sqrt{x}(\sqrt{x}-1)}{2} + r = \sqrt{x} + \sqrt{x} + r = 2\sqrt{x} + r \checkmark$$

$$rf(a) - rf(-a) = f(a^2 - a) \xrightarrow{x^2} f(a) - f(-a) = 1 + a^2 - r$$

$$rf(a) - rf(-a) = f(a^2 + a) \xrightarrow{x^2} f(a) - f(-a) = 1 + a^2 + r$$

$$\rightarrow -\Delta f(a) = r \cdot a^2 + a \Rightarrow \boxed{f(a) = \frac{ra^2 + a}{-\Delta}} \checkmark$$

$$(a+r)f(a) - rf(a+r) = f(a^2 - ma + r) - 1$$

$$\downarrow a = -r : f(a) = 1 + r + r - 1$$

$$\downarrow a = 0 : rf(a) = r - 1 \xrightarrow{x^2} f(a) = r - 1 \Rightarrow r - 1 = 1 + r - 1 \Rightarrow r = 1$$

$$\Rightarrow f(a) = 1 \Rightarrow m = \frac{1}{r} = 1 \Rightarrow rf(a) = r - 1 = 1 - 1 = 0 \Rightarrow f(a) = 0$$

$$f(a) + f\left(\frac{1}{a}\right) = \frac{ra^2 - 1 + r}{a} \rightarrow a + b + \frac{a}{a} + b = ra - 1 + \frac{r}{a}$$

$$\Rightarrow a = r, b = -1 \Rightarrow b = -r$$

$$\Rightarrow f(a) = ra - r \Rightarrow f(-1) = -r - r = \boxed{-2r} \checkmark$$