Subject.
Date.

تکلیف شماره ۴ یازدهم دبیران B — امیر یوسف رزاق

① الف) $f(x) = \sqrt{\dfrac{x-1}{x} - \dfrac{x}{x-1}}$

شرط ۱ $\to x \neq 0$
شرط ۲ $\to x \neq 1$
شرط ۳ $\to \dfrac{x-1}{x} - \dfrac{x}{x-1} \geq 0$

$\Rightarrow \dfrac{x-1}{x} - \dfrac{x}{x-1} \geq 0 \xRightarrow{\text{مخرج مشترک}} \dfrac{(x-1)^2 - x^2}{x^2 - x} \geq 0 \Rightarrow \dfrac{1-2x}{x(x-1)} \geq 0$

	0		$0/5$		1	
$+$	ϕ	$-$	0	$+$	ϕ	$-$

ریشه‌ها: $\dfrac{1}{2} = 0/5$

$\Rightarrow$ شرط ۳ $\to (-\infty, 0) \cup [0/5, 1)$

$① \cap ② \cap ③ \Rightarrow D_f = (-\infty, 0) \cup [0/5, 1)$

ب) $f(x) = \dfrac{\dfrac{1}{x+1} - \dfrac{2}{x}}{\dfrac{3}{x-1} + \dfrac{1}{x+2}} \Rightarrow$

شرط ۱ $\to x \neq -1$
شرط ۲ $\to x \neq 0$
شرط ۳ $\to x \neq 1$
شرط ۴ $\to x \neq -2$
شرط ۵ $\to \dfrac{3}{x-1} + \dfrac{1}{x+2} \neq 0 \xRightarrow{\text{مخرج مشترک}} \dfrac{3x+6+x-1}{(x-1)(x+2)} \neq 0$

$\Rightarrow \boxed{\dfrac{4x+5}{(x-1)(x+2)} \neq 0}$

شرط ۵
$\Rightarrow 4x + 5 \neq 0 \Rightarrow \boxed{x \neq \dfrac{-5}{4}}$

$① \cap ② \cap ③ \cap ④ \cap ⑤$

$\boxed{\Rightarrow D_f = \mathbb{R} - \left\{-2, \dfrac{-5}{4}, -1, 0, 1\right\}}$

شرط ۱ $\to \left(\left(\tfrac{1}{3}\right)^x - 9\right)\left(32 - 4^x\right) \geq 0$

② الف) $f(x) = \sqrt{\left(\left(\tfrac{1}{3}\right)^x - 9\right)\left(32 - 4^x\right)}$

$\left(\tfrac{1}{3}\right)^x - 9 = 0 \Rightarrow \left(\tfrac{1}{3}\right)^x = 3^2 = \tfrac{1}{3}^{-2} \Rightarrow \boxed{x = -2}$

$32 - 4^x = 0 \Rightarrow 2^5 - 4^x = 0 \Rightarrow 2^5 = 4^x = 2^{2x} \Rightarrow 2x = 5 \Rightarrow \boxed{x = \dfrac{5}{2}}$

	-2		$\frac{5}{2}$	
$+$	ϕ	$-$	ϕ	$+$

$\Rightarrow \boxed{D_f = (-\infty, -2] \cup [\tfrac{5}{2}, +\infty)}$

ب) $\sqrt[4]{x-1} + \sqrt{y+1} = 3 \to$ شرط ۱ $x - 1 \geq 0 \Rightarrow \boxed{x \geq 1}$
شرط ۲ $y + 1 \geq 0$

$\sqrt{y+1} = 3 - \sqrt[4]{x-1} \Rightarrow 3 - \sqrt[4]{x-1} \geq 0 \Rightarrow \sqrt[4]{x-1} \leq 3 \Rightarrow x - 1 \leq 3^4 = 81$

$\Rightarrow x - 1 \leq 81 \Rightarrow \boxed{x \leq 82}$ شرط دوم

$① \cap ② \Rightarrow D_f = \boxed{[1, 82]}$

PAPCO

Subject.
Date.

③ $\dfrac{\log_{\text{۲}}(x^2-x-2)}{\sqrt{x^2-1}+1}$ $\Rightarrow$ ① شرط $(x^2-x-2)>0$
② شرط $x^2-1\geq 0$

مخرج هیچگاه صفر نمی‌شود ← نامنفی

$x^2-x-2=(x+1)(x-2) \Rightarrow$ ریشه‌ها $= \begin{matrix}-1\\2\end{matrix} \Rightarrow$ $\begin{matrix} & -1 & & 2 & \\ + & \phi & - & \phi & + \end{matrix}$

$\boxed{(-\infty,-1)\ ,\ (2,+\infty)}$ ①

$x^2-1\geq 0 \Rightarrow (x-1)(x+1) \Rightarrow$ ریشه‌ها $= \begin{matrix}-1\\+1\end{matrix} \Rightarrow$ $\begin{matrix} & -1 & & 1 & \\ + & \phi & - & \phi & + \end{matrix}$

$\Rightarrow \boxed{(-\infty,-1]\cup[1,+\infty)}$ ② $\boxed{①\cap② \Rightarrow D_f=(-\infty,-1)\cup(2,+\infty)}$

④ $\sqrt{3+ax-x^2}$ در بازه $\Rightarrow [-2,b]$, $a+b=?$

$x^2 \rightarrow$ منفی در زیر رادیکال $\Rightarrow D=[x,y]$ (x و y ریشه‌ها هستند) $\Rightarrow D_{\text{تابع}}=[-2,b]$

$\Rightarrow -2, b =$ ریشه‌ها , $x=-2 \Rightarrow 3-2a-4=0 \Rightarrow -1-2a=0$

$\Rightarrow -2a=1 \Rightarrow a=\frac{-1}{2} \Rightarrow x^2-\frac{1}{2}x+3=0 \Rightarrow \begin{cases} x_1=-2 \\ x_2=\frac{3}{2} \Rightarrow b=\frac{3}{2}\end{cases}$

$\Rightarrow a+b=-\frac{1}{2}+\frac{3}{2}=1$

⑤ $f(x)=\begin{cases} 3x-2\ , & x\geq 1 \\ 2x+3\ , & x<1\end{cases}$ $D_g=?$ $\left(g(x)=\sqrt{f(x)-x}\right)$

شرط ۱ $\rightarrow f(x)-x\geq 0$

$f(x)=3x-2 \Rightarrow f(x)-x\geq 0 \Rightarrow 3x-2-x\geq 0 \Rightarrow 2x-2\geq 0$

$\Rightarrow 2(x-1)\geq 0 \Rightarrow x\geq 1$ ① , $f(x)=2x+3 \Rightarrow f(x)-x\geq 0$

$\Rightarrow 2x+3-x\geq 0 \Rightarrow x+3\geq 0 \Rightarrow x\geq -3 \Rightarrow -3\leq x\leq 1$ ②

$①\cap②=[1,+\infty)\cup[-3,1) \Rightarrow \boxed{D_g=[-3,+\infty)}$

PAPCO

Scanned by TapScanner

Subject.

Date.

(6) $f(x) = \begin{cases} (a+1)(x+2) & : x > 1 \\ 3a + 2x & : x \leq 1 \end{cases}$

$2f(5) = f(-2) + a$

$a = ?$

$x = -2 \rightarrow f(-2) = 3a + \boxed{2(-2) = 3a - 4}$

$x = 5 \rightarrow \boxed{2f(5) = 2 \times 7(a+1) = 14a + 14}$, $2f(5) = f(-2)$

$\Rightarrow 3a - 4 + a = 14a - 14 \Rightarrow 4a - 4 = 14a - 14 \Rightarrow 10a = -18$

$\Rightarrow \boxed{a = \frac{-18}{10} = -1.8}$ $\cancel{4}$ ✓

(7) $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2$, $f(2-\sqrt{3}) + f(2+\sqrt{3}) = ?$

$f(x) + f\frac{1}{x} = (\sqrt{x} + 2 + \frac{1}{\sqrt{x}}) + (\frac{1}{\sqrt{x}} + \sqrt{x} + 2) = 2(\sqrt{x} + \sqrt{\frac{1}{x}}) + 4$

$x = 2 + \sqrt{3} \Rightarrow f(2+\sqrt{3}) + f(2-\sqrt{3}) = 2(\sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}}) + 4$

$y^2 = (\sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}})^2 = 2 + \sqrt{3} + 2 - \sqrt{3} + \sqrt{(2+\sqrt{3})(2-\sqrt{3})} \times 2$

$\Rightarrow y^2 = 2 + 2 + 2\sqrt{1} = 3(2) = 6 \Rightarrow y^2 = 6 \Rightarrow y = \sqrt{6} = (\sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}})$

$f(2+\sqrt{3}) + f(2-\sqrt{3}) = 2(\sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}}) + 4$

$\Rightarrow f(2+\sqrt{3}) + f(2-\sqrt{3}) = \boxed{2\sqrt{6} + 4}$

(8) $2f(x) - 3f(-x) = 4x^2 - x$, $f(x) = ?$

$2f(x) - 3f(-x) = 4x^2 - x \Rightarrow \begin{cases} 4f(x) - 6f(-x) = 8x^2 - 2x \\ 6f(-x) - 9f(x) = 12x^2 + 3x \end{cases}$

$2f(-x) - 3f(x) = 4x^2 + x \Rightarrow$

$-5f(x) = 20x^2 + x \Rightarrow$ دستگاه

$\boxed{f(x) = -4x^2 - \frac{x}{5}}$

9

$(x+2)\,f(x) - 3x f(x+2) = 4x^2 - mx + 3m - 1$

$f(0) = ?$

$x = -2 \rightarrow (-2+2)\,f(-2) + 6f(0) = 16 + 2m + 3m - 1 \Rightarrow$

$6f(0) = 16 + 5m - 1 \Rightarrow \boxed{6f(0) = 15 + 5m} \Rightarrow$

$x = 0 \rightarrow (0+2)\,f(0) - 3(0)\,f(2) = 3m - 1 \Rightarrow 2f(0) = 3m - 1 \Rightarrow 6f(0) = 9m - 3$

$$\begin{cases} 6f(0) = 5m + 15 \\ 6f(0) = 9m - 3 \end{cases} \Rightarrow 9m - 3 = 5m + 15 \Rightarrow 4m = 18 \Rightarrow m = \frac{9}{2}$$

$\Rightarrow 6f(0) = 5m + 15 \Rightarrow 6f(0) = \frac{45}{2} + 15$

$\Rightarrow 2f(0) = \frac{\frac{75}{2}}{\frac{3}{1}} = \boxed{\frac{75}{4} = f(0)}$

10

$f \rightarrow$ تابع خطی ، $f(x) + f(\frac{1}{x}) = \frac{3x^2 - 12x + 3}{x}$ ، $f(-1) = ?$

$f(x) \rightarrow$ تابع خطی $\Rightarrow f(x) + f(\frac{1}{x}) = ax + b + \frac{a}{x} + b = ax + \frac{a}{x} + 2b$

$\Rightarrow 2f(x) + f(\frac{1}{x}) = \frac{3x^2 - 12x + 3}{x} \Rightarrow \frac{3x^2 - 12x}{x} + \frac{3}{x} \Rightarrow 3x + \frac{3}{x} - 12$

$\Rightarrow a = 3,\ 2b = -12 \Rightarrow b = -6 \Rightarrow f(x) = ax + b \Rightarrow f(x) = 3x - 6$

$\Rightarrow f(-1) = 3(-1) - 6 \Rightarrow f(-1) = -3 - 6 = -9 \Rightarrow \boxed{f(-1) = -9}$