

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{x^2 - 2x + 1 - x^2}{x(x-1)} \geq 0$
 $\frac{1}{1+\frac{1}{x}} - \frac{1}{\frac{1}{x}+1} \rightarrow D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{1}{x}}{\frac{1}{x-1} + \frac{1}{x+2}} \rightarrow \frac{\frac{1}{x+1} - \frac{1}{x}}{\frac{1}{x-1} + \frac{1}{x+2}} \rightarrow x+2 \neq 0 \rightarrow x \neq -2$
 $x \neq 0$
 $x+1 \neq 0 \rightarrow x \neq -1$
 $x-1 \neq 0 \rightarrow x \neq 1$
 $D_f = \mathbb{R} - \{0, -1, -2, 1, -\frac{5}{2}\}$

الف) $f(x) = \sqrt{((\frac{1}{x})^x - 9)(32 - x^x)} \rightarrow \frac{-2}{1+\frac{1}{x}} - \frac{\frac{5}{2}}{\frac{1}{x}+1}$
 $D_f = (-\infty, -2] \cup [\frac{5}{2}, +\infty)$

ب) $\sqrt{x-1} + \sqrt{y+1} = 3$
 I. $x-1 \geq 0 \rightarrow x \geq 1$
 II. $y+1 \geq 0 \rightarrow y \geq -1$
 $\sqrt{y+1} = 3 - \sqrt{x-1} \rightarrow \sqrt{y+1} \geq 0 \rightarrow 3 - \sqrt{x-1} \geq 0 \rightarrow \sqrt{x-1} \leq 3 \rightarrow x-1 \leq 9 \rightarrow x \leq 10$
 $D_f = [1, 10]$

$f(x) = \frac{\log x}{\sqrt{x^2-1} + 1}$

I. $x^2-1 \geq 0 \rightarrow x \geq 1, x \leq -1$
 II. $x^2-x-2 \geq 0 \rightarrow \frac{-1}{1+\frac{1}{x}} - \frac{2}{\frac{1}{x}+1}$
 III. $\sqrt{x^2-1} + 1 \neq 0$
 $I \cap II \cap III \rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$

$f(x) = \sqrt{3+ax-x^2} \rightarrow D_f = [-2, b]$
 $f(-2) = 0 \rightarrow 3 - 2a - 4 = 0 \rightarrow 2a = -1 \rightarrow a = -\frac{1}{2}$

$f(x) = \sqrt{3 - \frac{1}{x}x - x^2} \rightarrow 3 - \frac{1}{x}x - x^2 \geq 0 \rightarrow 3 - 1 - x^2 \geq 0 \rightarrow 2 - x^2 \geq 0 \rightarrow x^2 \leq 2 \rightarrow -\sqrt{2} \leq x \leq \sqrt{2}$
 $\frac{-2}{1+\frac{1}{x}} - \frac{\frac{3}{2}}{\frac{1}{x}+1} \rightarrow D_f = [-2, \frac{3}{2}]$

$a+b=1$

$g(x) = \sqrt{f(x)-x}$

$D_f = [-3, 1) \cup [1, +\infty) \rightarrow [-3, +\infty)$
 $f(x)-x \geq 0$

$f(x) - x =$

$f(x) - x =$

$3x-2-x = 2x-2 \rightarrow 2x-2 \geq 0 \rightarrow x \geq 1$

$3x+3-x = x+3 \rightarrow x+3 \geq 0 \rightarrow x \geq -3$

$\rightarrow -3 \leq x < 1$ II

$$f(n) = \begin{cases} (a+1)(n+2); & n > 1 \\ 3a+2n & ; n < 1 \end{cases}$$

$$f(a) = (a+1) \times 2 = 2(a+1)$$

$$f(-2) = 3a - 4$$

$$2f(a) = f(-2) + a$$

$$1(2a+2) = 3a - 4 + a \rightarrow 10a = -18$$

$$a = -1/5$$

$$f(n) = \sqrt{n} + \frac{1}{\sqrt{n}} + 2$$

$$f(\sqrt{2}-\sqrt{3}) + f(\sqrt{2}+\sqrt{3}) = \sqrt{\sqrt{2}-\sqrt{3}} + \frac{1}{\sqrt{\sqrt{2}-\sqrt{3}}} + 2 + \sqrt{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{\sqrt{2}+\sqrt{3}}} + 2 =$$

$$\rightarrow = \frac{\sqrt{2}-1}{\sqrt{\sqrt{2}-\sqrt{3}}} + \frac{\sqrt{2}}{\sqrt{\sqrt{2}-\sqrt{3}}} + \frac{\sqrt{2}+1}{\sqrt{\sqrt{2}+\sqrt{3}}} + \frac{\sqrt{2}}{\sqrt{\sqrt{2}+\sqrt{3}}} + 4 =$$

$$\rightarrow = \frac{2\sqrt{2}}{\sqrt{2}} + \frac{\sqrt{2}(\sqrt{2}+1) + \sqrt{2}(\sqrt{2}-1)}{\sqrt{2}-1} + 4 = \frac{2\sqrt{2}}{\sqrt{2}} + \frac{2\sqrt{2}}{\sqrt{2}-1} + 4 = 2\sqrt{2} + 4$$

$$2f(n) - 3f(-n) = 5n^2 - n \xrightarrow{\times 2} 4f(n) - 6f(-n) = 10n^2 - 2n$$

$$2f(-n) - 3f(n) = 5n^2 + n \xrightarrow{\times 3} 6f(-n) - 9f(n) = 15n^2 + 3n$$

$$-5f(n) = 5n^2 + n$$

$$f(n) = \frac{5n^2 + n}{-5}$$

$$(n+2)f(n) - 3nf(n+2) = 5n^2 - mn + 3m - 1$$

$$\downarrow$$

$$n = -2 : 4f(0) = 14 + 2m + 3m - 1 = 5m + 13$$

$$n = 0 : 2f(0) = 3m - 1 \xrightarrow{\times 2} 4f(0) = 6m - 2$$

$$\left\{ \begin{array}{l} 5m + 13 = 6m - 2 \\ \downarrow \\ 15 = m \end{array} \right.$$

$$m = 15$$

$$2f(0) = 14/2 \rightarrow f(0) = 7$$

$$f(n) = an + b$$

$$f(n) + f(1/n) = \frac{3n^2 - 12n + 3}{n} \rightarrow an + b + \frac{a}{n} + b = 3n + \frac{3}{n} - 12$$

$$\downarrow$$

$$\left\{ \begin{array}{l} a = 3 \\ 2b - 12 = -9 \end{array} \right. \rightarrow b = 1.5$$

$$f(n) = 3n - 9 \xrightarrow{f(-1)} f(-1) = -3 - 9 = -12$$