نام و نام خانوادگی: سیدرضا الهامیان   ۲۰   کلاس: یازدهم تجربی B   شمارهٔ تکلیف: ۲۷

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۱ (۲)

$$S_{\text{مثلث}} = \frac{1}{2} ab \sin\alpha \Rightarrow \frac{1}{2} \times 6 \times \sqrt{3} \times \sin\alpha = \frac{9}{2} \Rightarrow \sin\alpha = \frac{9}{6\sqrt{3}} \Rightarrow \frac{3}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin 60, \sin 120 = \frac{\sqrt{3}}{2} \Rightarrow \alpha = 60,\ \alpha = 120 \Rightarrow \frac{120}{60} = 2$$

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۵ (۲)

$$2\sin^2 x + \cos^2 x = \frac{4}{3} \Rightarrow \underbrace{\sin^2 x + \cos^2 x}_{1} + \sin^2 x = \frac{4}{3} \Rightarrow \sin^2 x = \frac{1}{3} \Rightarrow \sin x = \frac{\sqrt{3}}{3}$$

$$\sin^2 x + \cos^2 x = 1 \Rightarrow \frac{1}{3} + \cos^2 x = 1 \Rightarrow \cos = \sqrt{\frac{2}{3}} \Rightarrow \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

$$\Rightarrow \tan^2 x = \left(\frac{\frac{\sqrt{3}}{3}}{\frac{\sqrt{3}\sqrt{2}}{3}}\right)^2 = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$$

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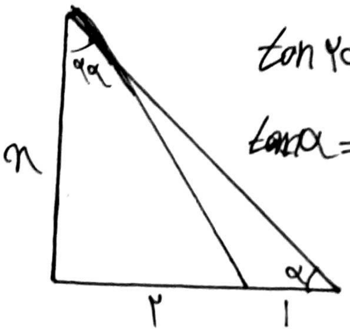
۳ (۲)



$$\tan 2\alpha = \frac{2}{x} = \frac{2\tan\alpha}{1-\tan^2\alpha} \Rightarrow \frac{\frac{2x}{3}}{1-\frac{x^2}{9}} = \frac{2}{x}$$

$$\tan\alpha = \frac{x}{3}$$

$$\Rightarrow x^2 = \frac{9}{4} \Rightarrow x = \frac{3}{2}$$

$$\Rightarrow \cot\alpha = \frac{3}{\frac{3}{2}} = 2$$

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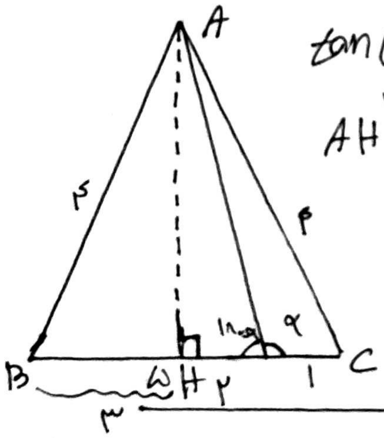
۴ (۲)



$$\tan(180-\alpha) = \frac{AH}{2} \Rightarrow \tan(180-\alpha) = \frac{\sqrt{7}}{2} \Rightarrow \tan(\alpha) = -\frac{\sqrt{7}}{2}$$

$$AH^2 + HC^2 = AC^2 \Rightarrow AH = \sqrt{16-9} = \sqrt{7}$$

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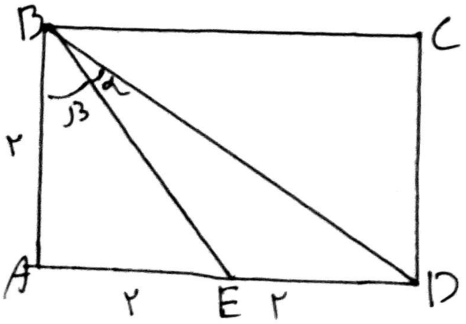
۲ (۲)



مثلث $A\hat{B}E$ قائم‌الزاویه است، و اضلاع قائم برابر است پس $\beta = 45°$

$$\Rightarrow \tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \times \tan\beta} \Rightarrow$$

$$2 = \frac{\tan\alpha + 1}{1 - \tan\alpha} \Rightarrow \tan\alpha = \frac{1}{3} \Rightarrow \cot\alpha = 3$$

$\sin^6\alpha + 4\cos^2\alpha \Rightarrow \sin^6\alpha + 4\cos^2\alpha + 4\sin^2\alpha - 4\sin^2\alpha \Rightarrow (\sin^2\alpha - 2)^2$

$\Rightarrow \frac{(\sin^2\alpha-2)^2}{1+(1-\sin^2\alpha)} - \frac{(\cos^2\alpha-2)^2}{1+(1-\cos^2\alpha)} = \frac{(2-\sin^2\alpha)^2}{2-\sin^2\alpha} - \frac{(2-\cos^2\alpha)^2}{2-\cos^2\alpha}$ (۲) ۶

$\Rightarrow 2-\sin^2\alpha + 2 + \cos^2 = \cot^2\alpha - \sin^2\alpha = \cos^2\alpha$ ✓

$\sin\left(\frac{9\pi}{2}+\alpha\right) = \cos\alpha \quad \cos\left(\frac{7\pi}{2}-\alpha\right) = -\sin\alpha \quad -\tan\left(\frac{3\pi}{2}-\alpha\right) = -\cot\alpha$

$(\cos\alpha \times -\sin\alpha) + \cot\alpha = -\frac{3}{5}\times\frac{4}{5} + \frac{3}{4} = -\frac{12}{25} + \frac{3}{4} = \frac{-48+75}{100} = \frac{27}{100}$ (۲) ۷

$\tan\alpha = \frac{4}{3} \Rightarrow 1+\tan^2 = \frac{1}{\cos^2\alpha}$

$\Rightarrow 1+\frac{16}{9} = \frac{1}{\cos^2\alpha} \Rightarrow \cos\alpha = \frac{3}{5}, \sin\alpha = \frac{4}{5}$ چون در ربع سوم است

$3\cos^4\left(\frac{2\pi}{12}\right) = 3\cos^4\frac{\pi}{3} = \frac{3}{4}, \quad \sqrt{2}(\sin\alpha\cos\alpha) = \sqrt{2}\left(-\sqrt{1-\sin^2\alpha}\right)$

$\Rightarrow \sqrt{2}\left(-\sqrt{1-\sin^2\frac{\pi}{6}}\right) = \sqrt{2}\times -\frac{1}{\sqrt{2}} = -1 \Rightarrow \frac{3}{4} - 1 = \frac{1}{4}$ ✓ (۲) ۵

$\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} \Rightarrow \frac{\frac{1}{2}}{1-\frac{1}{16}} = \frac{8}{15} = \frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha}$ صورت و مخرج بر $\cos\alpha$ تقسیم

(۲) ۹

$\Rightarrow \frac{\frac{\tan\alpha}{\cos\alpha} - \tan\alpha}{\tan\alpha - 1} \Rightarrow 1+\tan^2\alpha = \frac{1}{\cos^2} \Rightarrow 1+\frac{64}{225} = \frac{1}{\cos^2\alpha} \Rightarrow \cos\alpha = \frac{15}{17}$

$\frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{15}-1} = \frac{-16}{105}$ ✓

$\frac{\cot\alpha}{\sin\alpha} > 0 \Rightarrow \frac{\frac{\cos\alpha}{\sin\alpha}}{\sin\alpha} > 0 \Rightarrow \frac{\cos\alpha}{\sin^2\alpha} > 0 \Rightarrow 0 < \cos\alpha < 1 \Rightarrow$ ربع اول

$\Rightarrow 1-\cos\alpha > 0$

$2\sin\alpha - \sin 2\alpha < 0 \Rightarrow 2\sin\alpha - 2\sin\alpha\cos\alpha < 0 \Rightarrow 2\sin\alpha(1-\cos\alpha) < 0$ (۲) ۱۰

$\Rightarrow \sin\alpha < 0 \Rightarrow$ ربع سوم یا چهارم $\Rightarrow$ ربع چهار