

به نام خدا

سال نو پیشاپیش مبارک!
 Subject: _____
 Year: _____ Month: _____ Day: _____

page: () و نواحی با بانی - یازدهم بهر - 3 - تکلیف 27

با توجه به فرض سوال داریم:

$$S_{\triangle ABC} = \frac{1}{2} \times 4 \times \sqrt{3} \times \sin \alpha = 4\sqrt{3} \Rightarrow \sin \alpha = \frac{4\sqrt{3}}{3\sqrt{3}} = \frac{\sqrt{3}}{3}$$

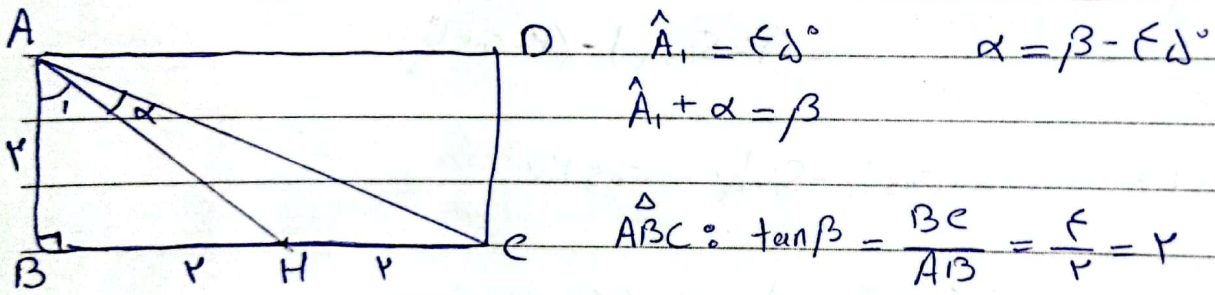
$$\Rightarrow \sin \alpha = \sin \frac{\pi}{3} \Rightarrow \begin{cases} \alpha = 2k\pi + \frac{\pi}{3} \xrightarrow{k=0} \alpha = \frac{\pi}{3} \\ \alpha = (2k+1)\pi - \frac{\pi}{3} \xrightarrow{k=0} \alpha = \frac{2\pi}{3} \end{cases}$$



چون $\alpha < \pi$ ، پس تنها جواب های قابل قبول $\frac{\pi}{3}$ و $\frac{2\pi}{3}$ هستند.

$$\frac{\frac{2\pi}{3}}{\frac{\pi}{3}} = \frac{120^\circ}{60^\circ} = 2 \quad \left. \vphantom{\frac{2\pi}{3}} \right\} \text{مقدار خواسته شده در سوال برابر است با:}$$

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$$\triangle ABC: \tan \beta = \frac{BE}{AB} = \frac{4}{2} = 2$$

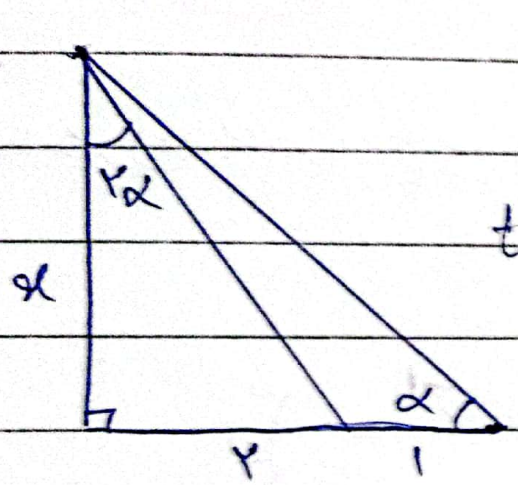


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$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} \Rightarrow \tan \alpha = \tan(\beta - 45^\circ)$$

$$= \frac{\tan \beta - \tan 45^\circ}{1 + \tan \beta \tan 45^\circ} = \frac{2-1}{1+2} = \frac{1}{3} \Rightarrow \cot \alpha = \frac{1}{\tan \alpha} = 3$$

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$$\tan 2\alpha = \frac{r}{q}$$

$$\tan \alpha = \frac{q}{r}$$

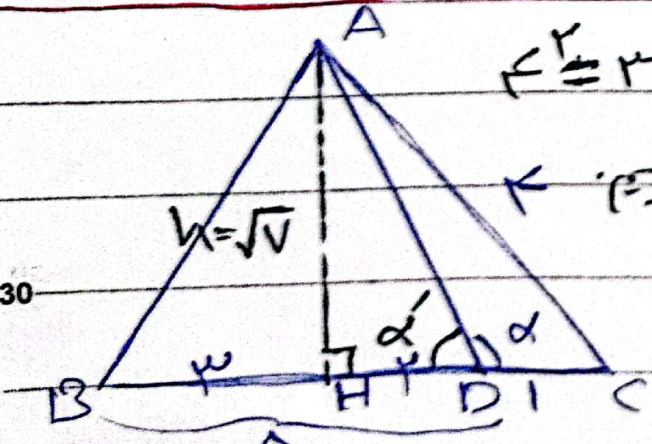
$$\tan 2\alpha = \frac{r \tan \alpha}{1 - \tan^2 \alpha} \Rightarrow \frac{r}{q} = \frac{\frac{r \cdot \frac{q}{r}}{r}}{1 - \frac{q^2}{r^2}} \Rightarrow \frac{r}{q} = \frac{q}{r - \frac{q^2}{r}}$$

$$\Rightarrow q r^2 = r^2 - q^2 \Rightarrow r^2 = r^2 - q^2 \Rightarrow q^2 = 0$$

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$$\Rightarrow r q = r^2 \Rightarrow q = \frac{r^2}{r} \Rightarrow \tan \alpha = \frac{\frac{r^2}{r}}{r} = \frac{1}{r} \Rightarrow \cot \alpha = \frac{1}{\tan \alpha} = r$$

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ارتفاع و دیرنگون را در $\triangle ABH$ حساب میکنیم: $r^2 = r^2 + h^2 \Rightarrow h = \sqrt{V}$
 اکنون در $\triangle AHD$ برای زاویه α' (مقابل α) $\tan \alpha' = \frac{AH}{HD}$

$$\tan \alpha' = \frac{AH}{HD} = \frac{\sqrt{V}}{r} \Rightarrow \tan \alpha = -\tan \alpha' = -\frac{\sqrt{V}}{r}$$

α' و α مکمل اند.

مقدار $\tan \alpha$ برابر $\frac{\sqrt{V}}{r}$ خواهد شد

$$r \sin^2 \alpha + \cos^2 \alpha = \frac{r}{p} \Rightarrow \sin^2 \alpha + \underbrace{\sin^2 \alpha + \cos^2 \alpha}_1 = \frac{r}{p}$$

$$\Rightarrow \sin^2 \alpha + 1 = \frac{r}{p} \Rightarrow \sin^2 \alpha = \frac{r}{p} - 1 \Rightarrow \cos^2 \alpha = \frac{r}{p}$$

$$\tan^2 \alpha = \frac{\sin^2 \alpha}{\cos^2 \alpha} = \frac{\frac{r}{p}}{\frac{r}{p}} = 1$$

$$\frac{\sin^2 \alpha + r \cos^2 \alpha}{1 + \cos^2 \alpha} = \frac{\cos^2 \alpha + r \sin^2 \alpha}{1 + \sin^2 \alpha} = \frac{\sin^2 \alpha - r \sin^2 \alpha + r}{r - \sin^2 \alpha} = \frac{\cos^2 \alpha - r \cos^2 \alpha + r}{r - \cos^2 \alpha}$$

$$\Rightarrow \frac{(r - \sin^2 \alpha)^2}{r - \sin^2 \alpha} = \frac{(r - \cos^2 \alpha)^2}{r - \cos^2 \alpha} = (r - \sin^2 \alpha) - (r - \cos^2 \alpha)$$

$$= r - \sin^2 \alpha - r + \cos^2 \alpha = \cos^2 \alpha - \sin^2 \alpha = \cos 2\alpha$$

$$\sin\left(\frac{9\pi}{p} + \alpha\right) = \sin\left(\pi + \frac{\pi}{p} + \alpha\right) = \sin\left(\frac{\pi}{p} + \alpha\right) = \cos \alpha$$

$$\cos\left(\frac{7\pi}{p} - \alpha\right) = \cos\left(2\pi + \frac{\pi}{p} - \alpha\right) = \cos\left(\frac{\pi}{p} - \alpha\right) = -\sin \alpha$$

$$\tan\left(\alpha - \frac{7\pi}{p}\right) = -\tan\left(\frac{7\pi}{p} - \alpha\right) = -\cot \alpha$$

$$\cos^2 \alpha = \frac{1}{1 + \tan^2 \alpha} = \frac{1}{1 + \frac{16}{9}} = \frac{9}{25} \Rightarrow \cos \alpha = \frac{3}{5}$$

$\cos \alpha < 0 \rightarrow$ $\sin \alpha > 0$ \rightarrow α \in $(\frac{\pi}{2}, \pi)$

$$\sin \alpha = \tan \alpha \cdot \cos \alpha = \frac{4}{3} \times \left(-\frac{3}{5}\right) = -\frac{4}{5} \Rightarrow \cot \alpha = \tan^{-1} = \frac{3}{4}$$

$$\sin\left(\frac{9\pi}{p} + \alpha\right) \cos\left(\frac{7\pi}{p} - \alpha\right) - \tan\left(\alpha - \frac{7\pi}{p}\right) = \cos \alpha (-\sin \alpha) + \cot \alpha$$

$$= \left(-\frac{3}{5}\right) \left(-\frac{4}{5}\right) + \frac{3}{4} = \frac{12}{25} + \frac{3}{4} = \frac{48}{100} + \frac{75}{100} = \frac{123}{100}$$

$$\mu \cos \alpha + \sqrt{r} \sin \alpha - \sqrt{r} \cos \alpha = \mu \cos \alpha + \sqrt{r} \frac{(\sin \alpha - \cos \alpha)}{\sqrt{r} \sin(\alpha - \frac{\pi}{4})}$$

$$= \mu \cos \alpha + r \sin(\alpha - \frac{\pi}{4}) \quad \alpha = \frac{\pi}{4} \rightarrow \mu \cos \frac{\pi}{4} + r \sin(-\frac{\pi}{4})$$

$$= \mu(\frac{1}{\sqrt{r}}) + r(\frac{-1}{\sqrt{r}}) = \frac{\mu}{\sqrt{r}} - 1 = \frac{1}{\sqrt{r}}$$



$$\sin \alpha = \frac{r \tan \frac{\alpha}{r}}{1 + \tan^2 \frac{\alpha}{r}} = \frac{r(\frac{1}{r})}{1 + (\frac{1}{r})^2} = \frac{1}{1+r}$$

$$\cos \alpha = \frac{1 - \tan^2 \frac{\alpha}{r}}{1 + \tan^2 \frac{\alpha}{r}} = \frac{1 - (\frac{1}{r})^2}{1 + (\frac{1}{r})^2} = \frac{10}{14}$$

$$\frac{\tan \alpha - \sin \alpha}{\sin \alpha - \cos \alpha} \times \frac{\cos \alpha}{\cos \alpha} = \frac{\sin \alpha - \sin \alpha \cos \alpha}{\sin \alpha \cos \alpha - \cos^2 \alpha} = \frac{\frac{1}{14} - \frac{1}{14} \times \frac{10}{14}}{\frac{1}{14} \times \frac{10}{14} - (\frac{10}{14})^2}$$



$$= \frac{1 \times 14 - 1 \times 10}{1 \times 10 - 10 \times 10} = \frac{1 \times 4}{-9 \times 10} = -\frac{4}{90}$$

$$\frac{\cot \alpha}{\sin \alpha} > 0 \Rightarrow \frac{\cos \alpha}{\sin \alpha} > 0 \Rightarrow \frac{\cos \alpha}{\sin^2 \alpha} > 0 \Rightarrow \sin^2 \alpha > 0 \Rightarrow \cos \alpha > 0$$

$$r \sin \alpha < \sin^2 \alpha \Rightarrow r \sin \alpha < r \sin \alpha \cos \alpha \Rightarrow \sin \alpha \cos \alpha - \sin \alpha > 0$$

$$\Rightarrow \sin \alpha (\cos \alpha - 1) > 0 \Rightarrow \sin \alpha < 0$$



$$\left. \begin{array}{l} \sin \alpha < 0 \\ \cos \alpha > 0 \end{array} \right\} \Rightarrow \alpha \text{ در ربع چهارم قرار دارد}$$