به نام خدا

۲۰

سال نو پیشاپیش مبارک !

Subject:

Year : Month : Day : ( )

۲۷ ونداحاجی باباکی - یازدهم پیسر B - تکلیف page:( )

با توجه به فرض سؤال داریم:

$$S_{\hat{ABC}} = \frac{۱}{۲}\times ۶\times\sqrt{۳}\times \sin\alpha = ۴{,}۵ \Rightarrow \sin\alpha = \frac{۴{,}۵}{۳\sqrt{۳}} = \frac{\sqrt{۳}}{۲}$$

$$\Rightarrow \sin\alpha=\sin\frac{\pi}{۳} \Rightarrow \begin{cases}\alpha = ۲k\pi+\frac{\pi}{۳} \xrightarrow{k=۰} \alpha=\frac{\pi}{۳}\\ \alpha=(۲k+۱)\pi-\frac{\pi}{۳}\xrightarrow{k=۰}\alpha=\frac{۲\pi}{۳}\end{cases}$$

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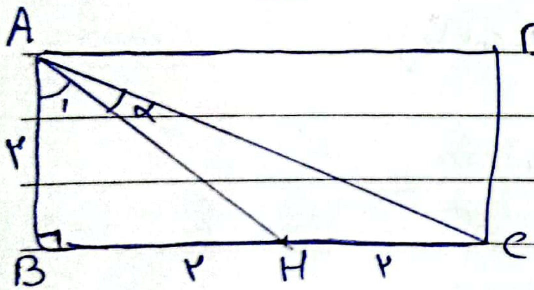
چون $۰<\alpha<\pi$ ، پس تنها جواب های قابل قبول $\frac{\pi}{۳}$ و $\frac{۲\pi}{۳}$ هستند.

مقدار خواسته شده در سؤال برابر است با: $\frac{۲\pi/۳}{\pi/۳} = \frac{۱۲۰^\circ}{۶۰^\circ} = ۲$



$\hat{A}_1 = ۴۵^\circ$

$\hat{A}_1 + \alpha = \beta$

$\alpha = \beta - ۴۵^\circ$

$\overset{\triangle}{ABC}: \tan\beta = \frac{BC}{AB} = \frac{۴}{۲} = ۲$

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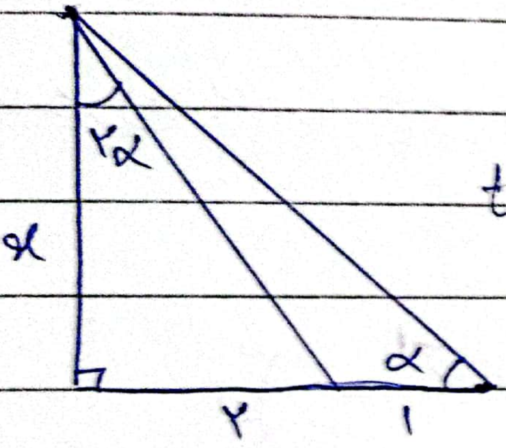
$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{۱+\tan\alpha\tan\beta} \Rightarrow \tan\alpha = \tan(\beta - ۴۵^\circ)$$

$$= \frac{\tan\beta - \tan ۴۵^\circ}{۱+\tan\beta\tan ۴۵^\circ} = \frac{۲-۱}{۱+۲} = \frac{۱}{۳} \Rightarrow \cot\alpha = \frac{۱}{\tan\alpha} = ۳$$

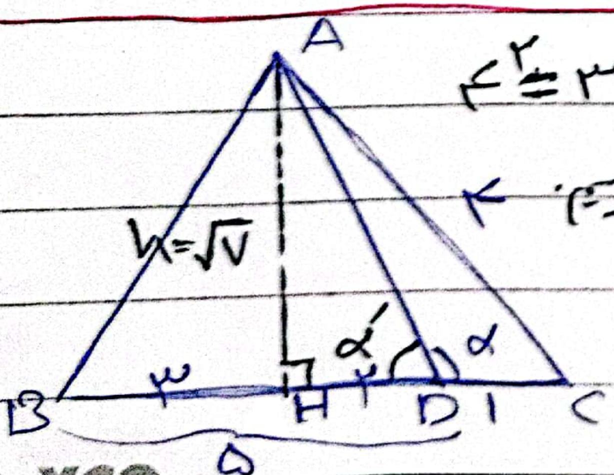


$\tan 2\alpha = \frac{2}{x}$ $\qquad$ $\tan\alpha = \frac{x}{3}$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} \Rightarrow \frac{2}{x} = \frac{2x/3}{1-x^2/9} \Rightarrow \frac{2}{x} = \frac{6x}{9-x^2}$$ ۲

$$\Rightarrow 6x^2 = 18 - 2x^2 \Rightarrow 8x^2 = 18 \Rightarrow 4x^2 = 9$$

$$\Rightarrow 2x = 3 \Rightarrow x = 3/2 \Rightarrow \tan\alpha = \frac{3/2}{3} = \frac{1}{2} \Rightarrow \cot\alpha = \frac{1}{\tan\alpha} = 2$$



ارتفاع وارد بر قاعده را در $A\hat{B}H$ حساب می‌کنیم: $4^2 = 3^2 + h^2 \Rightarrow h = \sqrt{7}$

اکنون در $A\hat{H}D$ برای زاویهٔ α' (مکمل α است) tan می‌نویسیم ←

$$\tan\alpha' = \frac{AH}{HD} = \frac{\sqrt{7}}{2} \Rightarrow \tan\alpha = -\tan\alpha' = -\frac{\sqrt{7}}{2}$$ ۲

α و α' مکمل اند.

مقدار $\tan\alpha$ برابر $\frac{\sqrt{7}}{2}$ خواهد شد

$$2\sin^2\alpha + \cos^2\alpha = \frac{4}{3} \Rightarrow \sin^2\alpha + \underbrace{\sin^2\alpha + \cos^2\alpha}_{1} = \frac{4}{3}$$

$$\Rightarrow \sin^2\alpha + 1 = \frac{4}{3} \to \sin^2\alpha = \frac{1}{3} \Rightarrow \cos^2\alpha = \frac{2}{3}$$

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$$\tan^2\alpha = \frac{\sin^2\alpha}{\cos^2\alpha} = \frac{1/3}{2/3} = \frac{1}{2}$$ ✓

$$\frac{\sin^4\alpha + 4\overbrace{\cos^2\alpha}^{1-\sin^2\alpha}}{1+\underbrace{\cos^2\alpha}_{1-\sin^2\alpha}} - \frac{\cos^4\alpha + 4\overbrace{\sin^2\alpha}^{1-\cos^2\alpha}}{1+\underbrace{\sin^2\alpha}_{1-\cos^2\alpha}} = \frac{\sin^4\alpha - 4\sin^2\alpha + 4}{2-\sin^2\alpha} - \frac{\cos^4\alpha - 4\cos^2\alpha + 4}{2-\cos^2\alpha}$$

$$\Rightarrow \frac{(2-\sin^2\alpha)^2}{2-\sin^2\alpha} - \frac{(2-\cos^2\alpha)^2}{2-\cos^2\alpha} = (2-\sin^2\alpha) - (2-\cos^2\alpha)$$ ۲ ۶

$$= 2 - \sin^2\alpha - 2 + \cos^2\alpha = \cos^2\alpha - \sin^2\alpha = \cos 2\alpha$$ ✓

$$\sin\left(\frac{9\pi}{2}+\alpha\right) = \sin\left(4\pi + \frac{\pi}{2} + \alpha\right) = \sin\left(\frac{\pi}{2}+\alpha\right) = \cos\alpha$$

$$\cos\left(\frac{7\pi}{2}-\alpha\right) = \cos\left(2\pi + \frac{3\pi}{2} - \alpha\right) = \cos\left(\frac{3\pi}{2}-\alpha\right) = -\sin\alpha$$

$$\tan\left(\alpha - \frac{3\pi}{2}\right) = -\tan\left(\frac{3\pi}{2}-\alpha\right) = -\cot\alpha$$ ۷

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$\cos\alpha < 0 \leftarrow$ چون α در ربع سوم است

$$\cos^2\alpha = \frac{1}{1+\tan^2\alpha} = \frac{1}{1+\frac{16}{9}} = \frac{9}{25} \Rightarrow \cos\alpha = \frac{-3}{5}$$

$$\sin\alpha = \tan\alpha \cdot \cos\alpha = \frac{4}{3} \times \left(\frac{-3}{5}\right) = \frac{-4}{5} \Rightarrow \cot\alpha = \tan^{-1}\alpha = \frac{3}{4}$$

$$\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right) - \tan\left(\alpha - \frac{3\pi}{2}\right) = \cos\alpha(-\sin\alpha) + \cot\alpha$$

$= \left(\frac{-3}{5}\right)\left(\frac{4}{5}\right) + \frac{3}{4} = 0{,}27$ ✓ :جواب نهایی

$$3\cos 4x+\sqrt{2}\sin x-\sqrt{2}\cos x=3\cos 4x+\sqrt{2}\underbrace{(\sin x-\cos x)}_{\sqrt{2}\sin(x-\pi/4)}$$

$$=3\cos 4x+2\sin\left(x-\frac{\pi}{4}\right)\xrightarrow{x=\frac{\pi}{12}}3\cos\frac{\pi}{3}+2\sin\left(-\frac{\pi}{6}\right)$$

$$=3\left(\frac{1}{2}\right)+2\left(\frac{-1}{2}\right)=\frac{3}{2}-1=\frac{1}{2}$$

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$$\sin\alpha=\frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}=\frac{2\left(\frac{1}{4}\right)}{1+\left(\frac{1}{4}\right)^2}=\frac{8}{17}\qquad \cos\alpha=\frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}=\frac{1-\left(\frac{1}{4}\right)^2}{1+\left(\frac{1}{4}\right)^2}=\frac{15}{17}$$

$$\frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha}\times\frac{\cos\alpha}{\cos\alpha}=\frac{\sin\alpha-\sin\alpha\cos\alpha}{\sin\alpha\cos\alpha-\cos^2\alpha}=\frac{\frac{8}{17}-\frac{8}{17}\times\frac{15}{17}}{\frac{8}{17}\times\frac{15}{17}-\left(\frac{15}{17}\right)^2}$$

$$=\frac{8\times17-8\times15}{8\times15-15\times15}=\frac{8\times2}{-7\times15}=\frac{-16}{105}$$

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$$\frac{\cot\alpha}{\sin\alpha}>0\Rightarrow\frac{\frac{\cos\alpha}{\sin\alpha}}{\sin\alpha}>0\Rightarrow\frac{\cos\alpha}{\sin^2\alpha}>0\xrightarrow{\sin^2\alpha>0}\boxed{\cos\alpha>0}$$

$$2\sin\alpha<\sin2\alpha\Rightarrow\cancel{2}\sin\alpha<\cancel{2}\sin\alpha\cos\alpha\Rightarrow\sin\alpha\cos\alpha-\sin\alpha>0$$

$$\Rightarrow\sin\alpha\underbrace{(\cos\alpha-1)}_{\text{منفی}\ <0}>0\Rightarrow\boxed{\sin\alpha<0}$$

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$$\left.\begin{array}{l}\sin\alpha<0\\ \cos\alpha>0\end{array}\right\}\Rightarrow$$ α در ناحیه چهارم قرار دارد