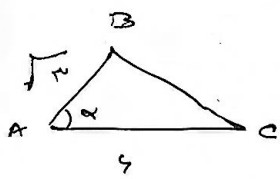


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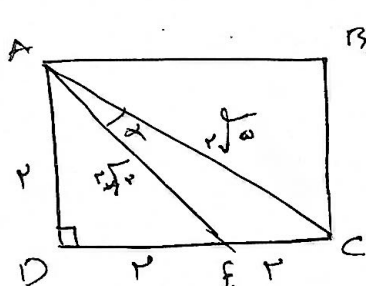
$$\rightarrow S_{\triangle ABC} = \frac{1}{2} AB \cdot AC \sin \alpha \rightarrow S_{\triangle ABC} = \frac{1}{2} \times \sqrt{2} \times 2 \times \sin \alpha$$

$$\rightarrow S_{\triangle ABC} = \frac{\sqrt{2}}{2} \rightarrow \alpha = \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\rightarrow \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1 \quad \checkmark$$

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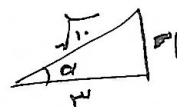


$$AE = \sqrt{2^2 + 1^2} = \sqrt{5}, \quad AC = \sqrt{2^2 + 2^2} = 2\sqrt{2}$$

$$\text{بقانون کوسین: } c^2 = (a+b)^2 - 2ab \cos \alpha$$

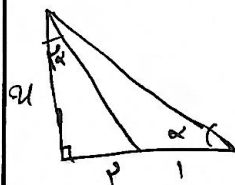
$$\rightarrow 4 = 2^2 + 5 - 2 \cdot 2 \cdot \sqrt{5} \cos \alpha \rightarrow \cos \alpha = \frac{2\sqrt{5}}{2\sqrt{5}} = \frac{2}{\sqrt{5}}$$

$$\Rightarrow \cos \alpha = \frac{2}{\sqrt{5}} \quad \checkmark$$



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$$\rightarrow \tan \alpha = \frac{a}{1}, \quad \tan 2\alpha = \frac{2}{a}$$

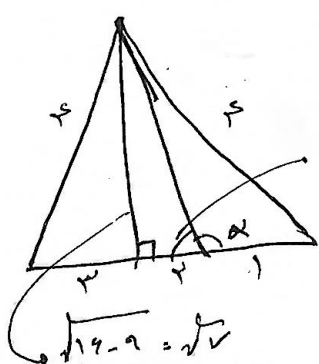
$$\rightarrow \tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha} = \frac{\frac{2a}{1}}{1 - \frac{a^2}{1}} = \frac{\frac{2a}{1}}{\frac{1-a^2}{1}} = \frac{2a}{1-a^2}$$

$$\rightarrow \frac{2a}{1-a^2} = \frac{2}{a} \Rightarrow 2a^2 = 1 - a^2 \Rightarrow 3a^2 = 1 \Rightarrow a = \frac{1}{\sqrt{3}} \quad (\tan \alpha)$$

$$\rightarrow \tan \alpha = \frac{1}{\sqrt{3}}, \quad \cos \alpha = \frac{\sqrt{3}}{2} \quad \checkmark$$

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$$\Rightarrow \tan(\pi - \alpha) = -\tan \alpha = \frac{\sqrt{5}}{2}$$

$$\Rightarrow \tan \alpha = \frac{-\sqrt{5}}{2} \quad \checkmark$$

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$$2 \sin^2 \alpha + \cos^2 \alpha = \sin^2 \alpha + \underbrace{\sin^2 \alpha + \cos^2 \alpha}_{=1} = \frac{5}{4} \rightarrow \sin^2 \alpha = \frac{1}{4}$$

$$\cos^2 \alpha = \frac{1}{\sin^2 \alpha} - 1 = 4 - 1 = 3 \Rightarrow \tan^2 \alpha = \frac{1}{3} \quad \checkmark$$

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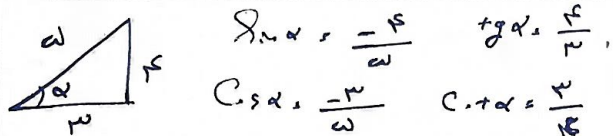
$$(\sin^2 - \cos^2)(\sin^2 + \cos^2) = (\sin^2 - \cos^2)$$

مخرج مشترك:  $\frac{\sin^2 \epsilon - \cos^2 \epsilon + \sin^2 \epsilon - \cos^2 \epsilon + \epsilon \cos^2 \epsilon - \epsilon \sin^2 \epsilon + \epsilon \cos^2 \epsilon \sin^2 \epsilon - \epsilon \cos^2 \epsilon \sin^2 \epsilon}{1 + \sin^2 \epsilon + \cos^2 \epsilon + \epsilon \sin^2 \epsilon}$

$(\sin^2 + \cos^2)^2 - \sin^2 \cos^2 = 1 - \sin^2 \cos^2$

$= \frac{(\sin^2 - \cos^2)^2 + \epsilon(\cos^2 - \sin^2) + (\sin^2 - \cos^2)(\sin^2 \epsilon + \cos^2 \epsilon + \sin^2 \cos^2 \epsilon)}{\epsilon + \sin^2 \cos^2 \epsilon} = \frac{-(\sin^2 - \cos^2)(\epsilon + \sin^2 \cos^2 \epsilon)}{\epsilon + \sin^2 \cos^2 \epsilon}$

$= \cos^2 \epsilon - \sin^2 \epsilon = \cos^2 \epsilon \checkmark$



$$\sin \alpha = \frac{w}{r} \quad \text{tg} \alpha = \frac{w}{v}$$

$$\cos \alpha = \frac{v}{r} \quad \text{c.t} \alpha = \frac{v}{w}$$

$\rightarrow \sin\left(\frac{9\pi}{4} + \alpha\right) \cos\left(\frac{5\pi}{4} - \alpha\right) - \text{tg}\left(\alpha - \frac{\pi}{4}\right) = (\cos \alpha)(-\sin \alpha) + \cos \alpha$

$= -\cos \alpha \sin \alpha + \cos \alpha = -\left(-\frac{v}{r}\right)\left(-\frac{w}{r}\right) + \frac{v}{r} = \frac{-vw}{r^2} + \frac{v}{r} = \frac{v}{r} \checkmark$

$$r \cos \frac{\pi}{6}, \quad r \cos \frac{\pi}{6} = r \cos \frac{\pi}{6} = \frac{r}{2}$$

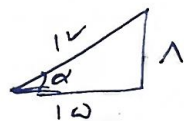
$$\sqrt{r} \sin \alpha - \sqrt{r} \cos \alpha = \sqrt{r} (\sin \alpha - \cos \alpha), \quad \sqrt{r} \left(\sqrt{r} \sin\left(\alpha - \frac{\pi}{2}\right)\right)$$

$$= r \sin\left(-\frac{\pi}{2}\right) = -r \sin \frac{\pi}{2} = -r$$

$\rightarrow (r \cos \frac{\pi}{6} + \sqrt{r} \sin \alpha - \sqrt{r} \cos \alpha) = \frac{r}{2} - r = -\frac{r}{2} \checkmark$

$$\text{tg}^2\left(\frac{\alpha}{2}\right) = \frac{1 - \cos \alpha}{1 + \cos \alpha} = \frac{1}{14} \rightarrow 14 - 14 \cos \alpha = 1 + \cos \alpha \rightarrow 14 = 15 \cos \alpha$$

$\Rightarrow \cos \alpha = \frac{14}{15} \rightarrow \cos \alpha > 0, \text{tg} \frac{\alpha}{2} > 0 \Rightarrow \alpha \text{ حاد}$



$$\frac{\text{tg} \alpha - \sin \alpha}{\sin \alpha - \cos \alpha} = \frac{\frac{1}{14} - \frac{1}{15}}{\frac{1}{15} - \frac{14}{15}} = \frac{\frac{1}{210}}{\frac{-13}{15}} = \frac{1 \times 15}{-13 \times 210} = \frac{15}{-2730} = \frac{-1}{182} \checkmark$$

$$r \sin \alpha < \sin^2 \alpha \rightarrow r \sin \alpha < r \sin \alpha \cos \alpha \rightarrow \sin \alpha < \sin \alpha \cos \alpha$$

$\rightarrow 0 < \sin \alpha \cos \alpha - \sin \alpha \rightarrow 0 < \sin \alpha (\cos \alpha - 1) \rightarrow \sin \alpha < 0$

$0 < \frac{\cos \alpha}{\sin \alpha} \xrightarrow{\sin \alpha < 0} \cos \alpha < 0 \Rightarrow \alpha \text{ حاد} \checkmark$