۱- (۲)

$\log_n m = a \Rightarrow n^a = m \qquad \log_{mn} m^2 n = b \Rightarrow mn^b = m^2 n \to (n^{a+1})^b = n^{2a+1}$

$b = \log_{mn} m^2 n = \log_{n^{a+1}} n^{2a+1} \to \log_n n \times \frac{2a+1}{a+1} = \frac{2a+1}{a+1} = b \qquad [b] = \boxed{1}$ ✓

$\Rightarrow [b] = \left[\frac{2a+1}{a+1}\right] = \left[\frac{2a+2-1}{a+1}\right] = \left[2 + \frac{-1}{a+1}\right] \xrightarrow{a>0} [1,\dots] = 1$

۲- (۲)

الف) $y = \sqrt{\frac{x}{\log_{\frac12} x}}$　1) $x > 0$　2) $\frac{x}{\log_{\frac12} x} \geq 0$

x		0		1	
x	$-$	0	$+$		$+$
$\log_{\frac12} x$			$+$	0	$-$
P			$+$		$-$

$\Rightarrow D_f = \boxed{(0,1)}$ ✓

ب) $y = \frac{\log_4 (x^2-x-2)}{\sqrt{x^2-1}+1}$

1) $x^2 - 1 \geq 0 \Rightarrow |x| \geq 1 \Rightarrow x \in (-\infty,-1] \cup [1,+\infty)$

2) $x^2 - x - 2 > 0 \xrightarrow{a+c=b} \begin{cases} x_1 = -1 \\ x_2 = 2 \end{cases}$

x		-1		2	
	$+$		$-$		$+$

$x \in (-\infty,-1) \cup (2,+\infty)$

$\xrightarrow{\text{اشتراک ۱ و ۲}} x \in \boxed{(-\infty,-1) \cup (2,+\infty)}$ ✓

۳- (۲)

$2\log_x a + \log_a \sqrt{x} = 2 \xrightarrow{x=9} 2\log_9 a + \log_a 3 = 2 \Rightarrow \log_3 a + \log_a 3 = 2 \xrightarrow{\log_3 a = t} t + \frac{1}{t} = 2$

$\Rightarrow t^2 - 2t + 1 = 0 \to t = 1 = \log_3 a \Rightarrow a = \boxed{3}$ ✓

۴- (۲)

$\log 2 \simeq 0,3 \Rightarrow \log 2 + \log 5 = 1 \Rightarrow \log 5 \simeq 0,7 \qquad \log 3 = 0,4$

$(\underbrace{\log 5}_{0,7} - \underbrace{\log 3}_{0,4})x^2 + (2\underbrace{\log 3}_{0,4})x - (\underbrace{\log 3}_{0,4} + \underbrace{\log 5}_{0,7}) = 0 \Rightarrow 0,3x^2 + 0,8x - 1,1 = 0$

اختلاف ریشه‌ها $= |\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{0,64 - 4(-0,33)}}{0,3} = \frac{\sqrt{0,64 + 1,32}}{0,3} = \frac{\sqrt{1,96}}{0,3} = \frac{1,4}{0,3} = \frac{14}{3} = 4\frac{2}{3}$ ✓

۵- (۲)

$\log_2 7 = 2,8 \qquad \log_5 2 = 0,5 \to 5^{\frac12} = 2 \Rightarrow 2^2 = 5 \qquad , 2^{\frac{28}{10}} = 7$

$\log_{14} 10 = \log_{7\times 2} 5\times 2 = \log_{2^{\frac{28}{10}}\times 2} 2^2 \times 2 = \log_{2^{\frac{38}{10}}} 2^3 = \frac{3}{\frac{38}{10}} \log_2 2 = \frac{30}{38} = \boxed{\frac{15}{19}}$ ✓

۶- (۱٫۷۵)

$\log_3 5 = 1,5 \quad , \log_2 3 = 1,6 \Rightarrow 2^{\frac{16}{10}} = 3 \qquad 3^{\frac{15}{10}} = 5 \Rightarrow (2^{\frac{16}{10}})^{\frac{15}{10}} = 5 \Rightarrow 2^{\frac{240}{100}} = 5 \Rightarrow 2^{\frac{24}{10}} = 5$

۲۶ دقت!

$\log_{15} 6 = \log_{3\times 5} 2\times 3 = \log_{2^{\frac{16}{10}}\times 2^{\frac{24}{10}}} 2\times 2^{\frac{16}{10}} = \log_{2^4} 2^{\frac{28}{10}} = \frac{\frac{28}{10}}{4}\log_2 2 = \frac{28}{40} = \boxed{\frac{7}{10}}$

۷- (۲)

$\log_8 18 = m \qquad \log_4 12 = ?$

$8^m = 18 \Rightarrow 2^{3m} = 2\times 3^2 \Rightarrow 2^{3m-1} = 3^2 \Rightarrow 2^{\frac{3m-1}{2}} = 3$

$\log_4 12 = \log_{2^2} 2^2 \times 3 = \log_{2^2} 2^2 \times 2^{\frac{3m-1}{2}} = \frac{\frac{3m+3}{2}}{2} \log_2 2 = \frac{3m+3}{4} = \frac{3}{4}(m+1)$ ✓

۸- (۲)

$(0,4)^{2x-1} = \left(\frac{125}{8}\right)^{x^2} \to \left(\frac{4}{10}\right)^{2x-1} = \left(\frac{5}{2}\right)^{3x^2} \Rightarrow \left(\frac{2}{5}\right)^{2x-1} = \left(\frac{2}{5}\right)^{-3x^2}$

$\to 2x - 1 = -3x^2 \to 3x^2 + 2x - 1 \xrightarrow{a+c=b} \begin{cases} x = -1 \\ x = \frac{-c}{a} = \frac{1}{3} \end{cases}$

اگر به جای x ها ۱- قرار دهیم چون جلوی لگاریتم منفی می‌شود داریم که =

$\log_8 9x+1 \xrightarrow{x=\frac13} \log_8 4 = \log_{2^3} 2^2 = \boxed{\frac{2}{3}}$ ✓

$\log_{k^m} a^n = \frac{n}{m}\log_k a$

۹- (۲)

$b = 8^{\frac{2}{3}(1+a)} = (2^3)^{\frac{2}{3}(1+a)} = 2^{2(1+a)} = 2^{2+2a} = 2^2 \times 2^{2a} = 4 \times 9 = 36$

$\log_2 3 = a \Rightarrow 2^a = 3 \Rightarrow 2^{2a} = 9$

$\log(36\times 3 - 8) = \log(100) = \boxed{2}$ ✓

10-

$$-4ax^2 + bx + \frac{1}{2}c \qquad \frac{b+c}{2} = a \Rightarrow 2a = b + c \Rightarrow b = 2a - c$$

$$\frac{1}{x_1 + x_2} = \frac{1}{S} = \frac{4a}{b} = \frac{4a}{2a-c} = \log 4 = 2\log 2 \Rightarrow \log 2 = \frac{2a}{2a-c} \Rightarrow \log_2 10 = \frac{2a-c}{2a}$$

$$\Rightarrow 1 - \frac{c}{2a} = \log_2 10 \Rightarrow 1 - \log_2 10 = \frac{c}{2a} \Rightarrow 2 - \log_2 100 = \frac{c}{a}$$

$$\Rightarrow \left(\frac{1}{\sqrt[8]{2}}\right)^{\frac{c}{a}} = \left(\sqrt[8]{\frac{1}{2}}\right)^{\frac{c}{a}} = \left(2^{\frac{-1}{8}}\right)^{\frac{c}{a}} = \left(2^{\frac{c}{a}}\right)^{\frac{-1}{8}} = \left(2^{2-\log_2 100}\right)^{\frac{-1}{8}} = \left(\frac{2^2}{2^{\log_2 100}}\right)^{\frac{-1}{8}} = \left(\frac{4}{100}\right)^{\frac{-1}{8}}$$

$$= \left(\frac{100}{4}\right)^{\frac{1}{8}} = (25)^{\frac{1}{8}} = \sqrt[4]{5}$$

(2)

6-

$$\log_{15} 6 = \frac{\log_3 6}{\log_3 15} = \frac{\log_3 2 + \log_3 3}{\log_3 3 + \log_3 5} = \frac{\frac{5}{8} + 1}{1 + 1{,}5} = \frac{\frac{13}{8}}{\frac{5}{2}} = \boxed{\frac{13}{20}}$$

$$\log_3 2 = \frac{1}{\log_2 3} = \frac{5}{8}$$