یازدهم پسر B تکلیف ۲۳ محمدامین جعفری

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$m = n^a$

$(mn)^b = m^b \cdot n^b = (n^a)^b \times n^b = n^{ab} \times n^b = n^{ab+b} = m^2 n = (n^a)^2 \times n = n^{2a} \times n = n^{2a+1}$

$2a+1 = ba+b \Rightarrow 2a+1 = b(a+1) \Rightarrow b = \frac{2a+1}{a+1} = 2 - \frac{1}{a+1} \Rightarrow [b] = \left[2 - \frac{1}{a+1}\right]$

$= 2 - \left[\frac{1}{a+1}\right] = 2$

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۲ الف)

$\log_{\frac{1}{2}} x \neq 0 \Rightarrow x \neq 1$

ب)

$x^2 - x - 2 > 0 \Rightarrow (x-2)(x+1) > 0$ $\begin{array}{ccccc} & -1 & & 2 & \\ + & | & - & | & + \end{array} \Rightarrow x \in (-\infty, -1) \cup (2, +\infty)$ ①

$\sqrt{x^2-1} + 1 \neq 0 \Rightarrow x \in \mathbb{R}$ ②

$x^2 - 1 \geqslant 0 \Rightarrow x^2 \geqslant 1 \Rightarrow x \in (-\infty, -1] \cup [1, +\infty)$ ③

$\xrightarrow{① \cap ② \cap ③} (-\infty, -1) \cup (2, +\infty)$

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$\Rightarrow 2\log_x a + \frac{1}{2}\log_a x = 2 \Rightarrow 2\log_x a + \frac{1}{2} \cdot \frac{1}{\log_x a} = 2$

$\Rightarrow 4(\log_x a)^2 - 4\log_x a + 1 \Rightarrow (2\log_x a - 1)^2 = 0 \Rightarrow 2\log_x a - 1 = 0$

$\Rightarrow 2\log_x a = 1 \Rightarrow \log_x a^2 = 1 \Rightarrow a^2 = x = 9 \Rightarrow a = \pm 3$

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$\frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{(\log 9)^2 - 4(\log \frac{5}{4})(\log 5)}}{\log \frac{5}{4}} \xrightarrow[\log \frac{5}{4} = \log 5 - \log 4]{\log 9 = 2\log 3 = 0.8,\ \log 5 = \log 4 + \log \frac{5}{4}} = \frac{\sqrt{(0.8)^2 - 4\left((\log 5)^2 - (\log 4)^2\right)}}{\log 5 - \log 4}$

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$m = \log_{۸}^{۱۸} = \frac{۲}{۳}\log_{۴}^{۱۸} = \frac{۲}{۳}(\log_{۴}^{۲} + \log_{۴}^{۹}) = \frac{۲}{۳}(\frac{۱}{۲} + \log_{۴}^{۹}) = \frac{۱}{۳} + \frac{۲}{۳}\log_{۴}^{۹}$

$\Rightarrow \log_{۴}^{۹} = \frac{۳}{۲}m - \frac{۱}{۲}$ $\quad \log_{۴}^{۱۲} = \log_{۴}^{۹} + \log_{۴}^{\frac{۴}{۳}} = \frac{۳}{۲}m - \frac{۱}{۲} + \log_{۴}^{۴} - \log_{۴}^{۳}$

$\xrightarrow{\log_{۴}^{۳} = \frac{۱}{۲}\log_{۴}^{۹}} \frac{۳}{۲}m - \frac{۱}{۲} + ۱ - \frac{۱}{۲}(\frac{۳}{۲}m - \frac{۱}{۲}) = \frac{۳}{۲}m + \frac{۱}{۲} - \frac{۳}{۴}m + \frac{۱}{۴} = \frac{۳}{۴}m + \frac{۳}{۴}$

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$(\frac{۴}{۱۰})^{۲x-۱} = (\frac{۱۲۵}{۸})^{x^۲} \Rightarrow (\frac{۲}{۵})^{۲x-۱} = ((\frac{۵}{۲})^{۳})^{x^۲} = (\frac{۵}{۲})^{۳x^۲} \Rightarrow (\frac{۲}{۵})^{۲x-۱} = (\frac{۲}{۵})^{-۳x^۲} \Rightarrow -۳x^۲ = ۲x-۱$

$\Rightarrow ۳x^۲ + ۲x - ۱ = ۰ \Rightarrow x = \begin{cases} -۱ \\ \frac{۱}{۳} \end{cases}$

$\Rightarrow \log_{۸}^{(۹x+۱)} = \begin{cases} \log_{۸}^{(-۹+۱)} = \log_{۸}^{-۸} \text{ غ ق ق} \\ \log_{۸}^{(\frac{۹}{۳}+۱)} = \log_{۸}^{۴} = \frac{۲}{۳}\log_{۲}^{۲} = \frac{۲}{۳} \end{cases}$

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$\frac{۲}{۳}(۱+a) = \frac{۲}{۳}(\log_{۲}^{۲} + \log_{۲}^{۳}) = \frac{۲}{۳}(\log_{۲}^{۶}) = \log_{۸}^{۳۶} = \log_{۸}^{b}$

$\Rightarrow b = ۳۶ \Rightarrow \log(۳(۳۶) - ۸) = \log ۱۰۰ = ۲$

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$\frac{۱}{\alpha + \beta} = \frac{۱}{\frac{-b}{-۸a}} = \frac{۸a}{b} = \log ۴ \quad \frac{b+c}{۲} = a$