

$$\log_m^n = a \Rightarrow \log_{mn}^{mn} = b \Rightarrow \log_{mn}^{mn} + \log_{mn}^m = b \Rightarrow 1 + \log_{mn}^m = b$$

$$\Rightarrow \log_{mn}^m = \frac{1}{\log_m^{mn}} \Rightarrow 1 + \frac{1}{\log_m^{mn}} = b \Rightarrow 1 + \frac{1}{\log_m^n + \log_m^n} = b \Rightarrow 1 + \frac{1}{1 + \log_m^n} = b$$

$$\Rightarrow \log_m^n = a \Rightarrow \log_m^n = \frac{1}{a} \Rightarrow 1 + \frac{1}{\frac{1}{a}} = b \Rightarrow 1 + \frac{1}{\frac{a+1}{a}} = b \Rightarrow [b] = \left[\frac{a}{a+1}\right] = 1$$

الف) $y = \frac{1}{\log_{\frac{1}{x}}^x} \Rightarrow \log_{\frac{1}{x}}^x \neq 0 \Rightarrow x \neq 1$ $x=0$ $\log_{\frac{1}{x}}^x = 0 \Rightarrow x=1$ $\log_{\frac{1}{x}}^x \Rightarrow x > 0$

$\frac{1}{\log_{\frac{1}{x}}^x} - \frac{1}{\log_{\frac{1}{x}}^x} + \frac{1}{\log_{\frac{1}{x}}^x} \Rightarrow \log_{\frac{1}{x}}^x \Rightarrow x < 0$ باید باشد $\Rightarrow D_f = 0 < x < 1$ ✓

ب) $y = \frac{\log_r(u^2 - u + 1)}{\sqrt{u^2 - 1} + 1} \Rightarrow \log_r(u^2 - u + 1) \Rightarrow u^2 - u + 1 > 0 \Rightarrow (u-2)(u+1) > 0 \Rightarrow \frac{1}{u-2} - \frac{1}{u+1} > 0$

$\Rightarrow \frac{1}{u-2} - \frac{1}{u+1} > 0 \Rightarrow \frac{u+1}{(u-2)(u+1)} > 0 \Rightarrow \frac{1}{u-2} > 0 \Rightarrow u > 2$ $\Rightarrow \frac{1}{u-2} > 0 \Rightarrow \frac{1}{u-2} > 0 \Rightarrow \frac{1}{u-2} > 0$ ✓

$$2 \log_a^x + \log_a^{\sqrt{x}} = 2 \Rightarrow x = 9 \Rightarrow 2 \log_a^x + \frac{1}{x} \log_a^x = 2 \Rightarrow 2 \log_a^x + \frac{1}{x} \log_a^x = 2$$

$$\Rightarrow \log_a^x = t \Rightarrow 2t + \frac{1}{t} = 2 \Rightarrow \frac{2t^2 + 1}{t} = 2 \Rightarrow 2t^2 - 2t + 1 = 0$$

$$\Rightarrow (2t-1)^2 = 0 \Rightarrow t = \frac{1}{2} \Rightarrow \log_a^x = \frac{1}{2} \Rightarrow x = 9 \Rightarrow x = 9^{\frac{1}{2}} \Rightarrow x = 3$$
 ✓

$$[\log_{\frac{1}{x}}^x] x^2 + (\log_{\frac{1}{x}}^x) x - \log_{\frac{1}{x}}^x = 0 \Rightarrow [\log_{\frac{1}{x}}^x - \log_{\frac{1}{x}}^x] x^2 + (\log_{\frac{1}{x}}^x) - \log_{\frac{1}{x}}^x = 0$$

$$[1 - \log_{\frac{1}{x}}^x] x^2 + (\log_{\frac{1}{x}}^x) - 1 + \log_{\frac{1}{x}}^x - \log_{\frac{1}{x}}^x = 0 \Rightarrow \log_{\frac{1}{x}}^x = \frac{1}{x^2}, \log_{\frac{1}{x}}^x = \frac{1}{x^2}$$

$$13x^2 + 18x - 11 = 0 \Rightarrow 3x^2 + 18x - 11 = 0 \Rightarrow (3x-3)(2x+11) = 0 \Rightarrow x = 1, x = -\frac{11}{2}$$

$$\Rightarrow |x - \beta| \Rightarrow |1 - (-\frac{11}{2})| \Rightarrow |x - \beta| = |\frac{15}{2}| \Rightarrow |x - \beta| = \frac{15}{2}$$
 ✓

$$\log_{\frac{1}{x}}^x = 211, \log_{\frac{1}{x}}^x = 0.1 \Rightarrow \log_{\frac{1}{x}}^x \Rightarrow \frac{\log_{\frac{1}{x}}^x}{\log_{\frac{1}{x}}^x} \Rightarrow \frac{\log_{\frac{1}{x}}^x + \log_{\frac{1}{x}}^x}{\log_{\frac{1}{x}}^x + \log_{\frac{1}{x}}^x}$$

$$\Rightarrow \frac{\log_{\frac{1}{x}}^x + 1}{\log_{\frac{1}{x}}^x + 1} \Rightarrow \log_{\frac{1}{x}}^x = \frac{1}{\omega} \Rightarrow \frac{1+1}{1+1} = \frac{1}{\omega} = \frac{1}{\omega} = \frac{1}{\omega} = \frac{1}{\omega}$$
 ✓

$$\log_{10}^4 = \frac{\log_v^4}{\log_v^{10}} \Rightarrow \frac{\log_v^4 + \log_v^4}{\log_v^4 + \log_v^4} \Rightarrow \log_v^4 \Rightarrow \frac{\log_v^4}{\log_v^4} \Rightarrow \log_v^4 = 114 \Rightarrow \frac{1}{\log_v^4} = 114$$

$$\Rightarrow \log_v^4 = \frac{10}{114} \Rightarrow \log_v^4 = \frac{10}{114} = 1/11.4 \Rightarrow \log_{10}^4 = \frac{1+11.4}{11.4+11.4} = 0.42 \checkmark$$

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$$\log_{\lambda}^{11} = m \Rightarrow \frac{1}{p} \log_{\lambda}^{11} = m \Rightarrow \log_{\lambda}^{11} = p m \Rightarrow \log_{\lambda}^p + p \log_{\lambda}^p = p m \Rightarrow \log_{\lambda}^p = \frac{p m - 1}{p}$$

$$\log_{\epsilon}^{11} \Rightarrow \frac{1}{p} \log_{\lambda}^{11} \Rightarrow \frac{1}{p} (\log_{\lambda}^{11} + p \log_{\lambda}^p) \Rightarrow \frac{1}{p} (p + \frac{p m - 1}{p}) \Rightarrow \log_{\epsilon}^{11} = \frac{p m + p}{p} \checkmark$$

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$$(1/15)^{10-1} = (\frac{1}{15})^{9} \Rightarrow (\frac{1}{15})^{9-1} = (\frac{1}{15})^{8} \Rightarrow p_{10}^9 = 1 - p_{10} \Rightarrow p_{10}^9 + p_{10} - 1 = 0 \Rightarrow (p_{10} + 1)(p_{10} - 1)$$

$$\Rightarrow p = -1 \quad \alpha = \frac{1}{p} \Rightarrow \log_{\lambda}^{9\alpha+1} \Rightarrow \alpha = -1 \Rightarrow 9(-1)+1 < 0 \Rightarrow \text{جواب صحیح}$$

$$\Rightarrow \alpha = \frac{1}{p} \Rightarrow \log_{\lambda}^{\epsilon} \Rightarrow \frac{p}{p} \log_{\lambda}^p = \frac{p}{p} \checkmark$$

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$$\log_{\lambda}^p = \alpha \quad \log_{\lambda}^b = \frac{p}{p}(1+\alpha) \Rightarrow \log_{\lambda}^b = \frac{p}{p}(1+\log_{\lambda}^p)$$

$$\Rightarrow \frac{1}{p} \log_{\lambda}^b = \frac{p}{p} (\log_{\lambda}^p) \Rightarrow \log_{\lambda}^b = p \log_{\lambda}^p \Rightarrow \log_{\lambda}^b = \log_{\lambda}^{p^4} \Rightarrow b = p^4 \checkmark$$

$$\Rightarrow \log_{\lambda}^{(pb-1)} \Rightarrow \log_{\lambda}^{(p^4 p^4 - 1)} \Rightarrow \log_{\lambda}^{100} = p \checkmark$$

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$$-10a^2 + 6a + \frac{1}{p}c = 0 \Rightarrow \frac{1}{a+p} = \log_{\lambda}^p \Rightarrow \frac{1}{-\frac{1}{a}} = \log_{\lambda}^p \Rightarrow -\frac{a}{1} = \log_{\lambda}^p \Rightarrow \frac{c}{b} = p \log_{\lambda}^p$$

$$\Rightarrow \frac{10a}{b} = \log_{\lambda}^p \Rightarrow c+b = 10a \Rightarrow \frac{c+b}{b} = \log_{\lambda}^p \Rightarrow \frac{c}{b} + 1 = 1 - \log_{\lambda}^p \Rightarrow \frac{c}{b} = -\log_{\lambda}^p$$

$$\Rightarrow \frac{c}{b} = -\log_{\lambda}^p \Rightarrow \frac{c}{10a} = -\frac{\log_{\lambda}^p}{\log_{\lambda}^p} \Rightarrow \frac{c}{10a} = -\log_{\lambda}^p \Rightarrow \frac{c}{a} = -10 \log_{\lambda}^p$$

$$\Rightarrow (\frac{1}{10})^{-10 \log_{\lambda}^p} \Rightarrow (10^{-1})^{-10 \log_{\lambda}^p} \Rightarrow (10)^{-10 \log_{\lambda}^p} \Rightarrow (10)^{\frac{1}{10}} \Rightarrow \sqrt[10]{10} \checkmark$$

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