نام و نام خانوادگی ...[illegible]... پاسخنامه تشریحی تکلیف شماره .... کلاس ....

$\log_{mn}^{m^2 n} = b \qquad \log_n^m = a \qquad \Big\vert\ \log_{mn}^{mn\times m} = 1+\log_{mn}^m$ $\log_{mn}^m = \frac{1}{\log_m^{mn}} = \frac{1}{1+\log_m^n} = \frac{1}{1+\frac{1}{\log_n^m}} = \frac{1}{1+\frac{1}{a}} = \frac{1}{\frac{a+1}{a}} = \frac{a}{a+1}$ $a>0 \Rightarrow \frac{a}{a+1} \in (0,1) \to 1+\frac{a}{a+1} \in (1,2) \to \left[1+\frac{a}{a+1}\right] = 1$	۱
الف) $\sqrt{\frac{x}{\log_{\frac12}^x}} \qquad \log_{\frac12}^x \Rightarrow x>0 \qquad \frac{x}{\log_{\frac12}^x} \geq 0 \ \begin{cases} 1>x>0 \to \log_{\frac12}^x>0 \Rightarrow \frac{x}{\log_{\frac12}^x}>0 \\ x>1 \to \log_{\frac12}^x<0 \Rightarrow \frac{x}{\log_{\frac12}^x}<0 \ \times \end{cases}$ $\frac{x}{\log_{\frac12}^x} \Rightarrow x \neq 1$ $D = (0,1)$ ب) $y = \frac{\log_2^{(x+1)(x-2)}}{\sqrt{x^2-1}+1} \qquad x^2-x-2>0 \qquad x^2-1 \geq 0 \to \begin{cases} x \geq 1 \ (3) \\ x \leq -1 \ (4) \end{cases} \qquad (1\cup 2)\cap(3\cup 4)$ $-1 \quad 2$ $(1,2) \to + \ \phi \ - \ \phi \ + \qquad \sqrt{x^2-1}+1 \neq 0 \to \sqrt{x^2-1} \neq -1 \qquad D_f = (-\infty,-1)\cup(2,\infty)$	۲
$2\log_9^a + \log_a^{\sqrt9} = \log_3^a + \log_a^3 = 2 \to \log_3^a + \frac{1}{\log_3^a} = 2$ $\log_3^a = x \to x+\frac{1}{x} = 2 \to x^2+1 = 2x \to x^2-2x+1 = 0 \to (x-1)^2 = 0$ $x = 1 \to \log_3^a = 1 \to a = 3^1 = 3$	۳
$\log\frac{5}{3} = \log 5 - \log 3 = \log\frac{10}{2} - \log 3 = \log 10 - \log 2 - \log 3 = 1-0.3-0.4 \approx 0.3$ $\log 9 = 2\log 3 = 2(0.4) = 0.8 \qquad \log 15 = \log 5 + \log 3 = \log 10 - \log 2 + \log 3 = 1.1$ $0.3x^2+0.8x-1.1 = 0 \to 3x^2+8x-11 = 0 \qquad \Delta = b^2-4ac \to 64-4(3)(-11) = 196$ $\lvert x_1-x_2 \rvert = \frac{\sqrt\Delta}{\lvert a \rvert} \to \frac{\sqrt{196}}{3} = \frac{14}{3}$	۴
$\log_2^5 = \frac{1}{\log_5^2} = \frac{1}{\frac{5}{10}} = 2 \qquad \log_2^5 + \log_2^2 = \log_2^{10} = 2+1 = 3$ $\log_2^3 + \log_2^4 = \log_2^{12} = 2.8+1 = 3.8$ $\frac{\log_2^{10}}{\log_2^{12}} = \frac{\log_2^{10}}{\log_2^{12}} = \frac{3}{3.8} = \frac{30}{38}$	۵

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۶

$\log_3 2 = \frac{1}{\log_2 3} = \frac{1}{\frac{16}{10}} = \frac{10}{16}$ $\qquad \log_3 2 + \log_3 3 = \log_3 6 = \frac{10}{16} + \frac{16}{16} = \frac{26}{16}$

$\log_3 5 + \log_3 3 = \log_3 15 = 1.5 + 1 = 2.5 = \frac{25}{10}$ $\qquad \frac{\log_3 6}{\log_3 15} = \frac{\frac{26}{16}}{\frac{25}{10}} = 0.65$

۷

$\log_{2^2} 2\times 3^3 = \log_{2^2} 2 + \log_{2^2} 3^3 = \frac{1}{2} + \frac{3}{2}\log_2 3 = m \to \frac{3}{2}\log_2 3 = m - \frac{1}{2}$

$\log_2 3 = \frac{2m-1}{3}$ (1) $\qquad \log_4 12 = \log_4 4 + \log_4 3 = 1 + \frac{1}{2}\log_2 3$ (2)

(1),(2) $\to 1 + \frac{3m-1}{4} = \frac{4+3m-1}{4} = \frac{3m+3}{4} = \frac{3}{4}(m+1)$

۸

$(\frac{4}{10})^{2m-1} = (\frac{2}{5})^{2m-1} = (\frac{5}{2})^{-2m+1}$

$(\frac{125}{8})^{m^2} = ((\frac{5}{2})^3)^{m^2} = (\frac{5}{2})^{3m^2}$

$\Big\} (\frac{5}{2})^{-2m+1} = (\frac{5}{2})^{3m^2} \to -2m+1 = 3m^2$

$3m^2 + 2m - 1 = 0$

$\Delta = 4 - 4(3)(-1) = 16$

$\frac{-2\pm 4}{6}$ $\begin{cases} \frac{1}{3} \Rightarrow (\frac{2}{5})^{-\frac{1}{3}} = (\frac{125}{8})^{\frac{1}{9}} \Rightarrow (\frac{5}{2})^{\frac{1}{3}} = (\frac{5}{2})^{\frac{1}{3}} \checkmark \\ -1 \to \log_8 -1 \ \times \end{cases}$

$\frac{1}{3} \to \log_8 4 \Rightarrow \log_{2^3} 2^2 = \frac{2}{3}\log_2 2 = \frac{2}{3}$

۹

$\log_2 b = 2(1+a)$ (1) $\quad \log_2 3 = a \to \log_2 2 + \log_2 3 = 1 + a = \log_2 6$

$2\log_2 6 = 2(a+1) \to \log_2 36 = 2(a+1)$ (2) $\qquad$ (1),(2) $\to 36 = b$

$\log(3(36)-8) = \log 100 = 2$

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$\frac{-b}{-4a} = \frac{b}{4a} \to \frac{4a}{b} = \log_2 4 \to b = \frac{4a}{\log_2 4}$ $\Big\}$ $2a = \frac{4a}{\log_2 4} + c$

$2a = b + c$

$c = 2a - \frac{4a}{\log_2 4} \Rightarrow \frac{c}{a} = 2 - \frac{4}{\log_2 4}$ $\qquad (\frac{1}{\sqrt[8]{2}})^{\frac{c}{a}} = (2^{-\frac{1}{8}})^{2 - \frac{4}{\log_2 4}} \to (2^{-\frac{1}{8}})^{2 - \frac{2}{\log_2 2}}$

$2^{(-\frac{1}{4} + \frac{1}{2\log_2 2})}$