نام و نام خانوادگی ....[illegible].......... پاسخنامه تشریحی تکلیف شماره .... کلاس ....

۱۸,۵

۱ (۲)

$\log_{mn} mn = b \qquad \log_n m = a \qquad \left|\ \log_{mn} mn \times m = 1+\log_{mn} m\right.$

$\log_{mn} m = \frac{1}{\log_m mn} = \frac{1}{1+\log_m n} = \frac{1}{1+\frac{1}{\log_n m}} = \frac{1}{1+\frac{1}{a}} = \frac{1}{\frac{a+1}{a}} = \frac{a}{a+1}$

$a>0 \Rightarrow \frac{a}{a+1} \in (0,1) \to 1+\frac{a}{a+1} \in (1,2) \to \left[1+\frac{a}{a+1}\right] = 1$ ✓

۲ (۲)

الف) $\sqrt{\frac{x}{\log_{\frac12} x}}$ $\qquad \log_{\frac12} x \Rightarrow x>0 \qquad \frac{x}{\log_{\frac12} x} \geq 0$ $\begin{cases} 1>x>0 \to \log_{\frac12} x>0 \Rightarrow \frac{x}{\log_{\frac12} x}>0 \\ x>1 \to \log_{\frac12} x<0 \Rightarrow \frac{x}{\log_{\frac12} x}<0 \end{cases}$ ✗

$\frac{x}{\log_{\frac12} x} \Rightarrow x \neq 1$

$D = (0,1)$ ✓

ب) $f(x) = \frac{\log_2 (x+1)(x-2)}{\sqrt{x^2-1}+1}$ $\qquad x^2-x-2>0 \qquad x^2-1 \geq 0 \to \begin{cases} x\geq 1 & (3) \\ x\leq -1 & (4) \end{cases}$ $\qquad (1\cup 2)\cap(3\cup 4)$

$(1,2)$ $\quad -1,\ 2:\ + - +$ $\qquad \sqrt{x^2-1}+1 \neq 0 \to \sqrt{x^2-1} \neq -1$ $\qquad D_f = (-\infty,-1)\cup(2,\infty)$ ✓

۳ (۲)

$2\log_9 a + \log_a \sqrt9 = \log_3 a + \log_a 3 = 2 \to \log_3 a + \frac{1}{\log_3 a} = 2$

$\log_3 a = x \to x+\frac{1}{x} = 2 \to x^2+1 = 2x \to x^2-2x+1=0 \to (x-1)^2=0$

$x=1 \to \log_3 a = 1 \to a = 3^1 = 3$ ✓

۴ (۲)

$\log\frac{5}{3} = \log 5 - \log 3 = \log\frac{10}{2} - \log 2 - \log 3 = 1 - 0.3 - 0.4 = 0.3$

$\log 9 = 2\log 3 = 2(0.4) = 0.8 \qquad \log 15 = \log 5 + \log 3 = \log 10 - \log 2 + \log 3 = 1.1$

$0.3x^2 + 0.8x - 1.1 = 0 \to 3x^2+8x-11=0 \qquad \Delta = b^2-4ac \to 64-4(3)(-11) = 196$

$|x_1-x_2| = \frac{\sqrt{\Delta}}{|a|} \to \frac{\sqrt{196}}{3} = \frac{14}{3}$ ✓

۵ (۲)

$\log_2 5 = \frac{1}{\log_5 2} = \frac{1}{\frac{5}{10}} = 2 \qquad \log_2 5 + \log_2 2 = \log_2 10 = 2+1 = 3$

$\log_2 7 + \log_2 2 = \log_2 14 = 2.8+1 = 3.8$

$\frac{\log_2 10}{\log_2 14} = \frac{\log 10}{\log 14} = \frac{3}{3.8} = \frac{30}{38}$ ✓

$\log_3 2 = \frac{1}{\log_2 3} = \frac{1}{\frac{16}{10}} = \frac{10}{16}$ $\quad \log_2 2 + \log_2 3 = \log_2 6 = \frac{10}{10} + \frac{16}{10} = \frac{26}{10}$ ۲ ۶

$\log_2 5 + \log_2 3 = \log_2 15 = 2.4 + 1.6 = 4.0 = \frac{40}{10}$ $\quad \frac{\log_2 6}{\log_2 15} = \frac{\frac{26}{10}}{\frac{40}{10}} = 0.65$ ✓

$\log_{2^3} 2\times 3^3 = \log_{2^3} 2 + \log_{2^3} 3^3 = \frac{1}{3} + \frac{3}{3}\log_2 3 = m \to \frac{3}{3}\log_2 3 = m - \frac{1}{3}$ ۲ ۷

$\log_2 3 = \frac{3m-1}{3}$ (1) $\quad \log_4 12 = \log_4 4 + \log_{2^2} 3 = 1 + \frac{1}{2}\log_2 3$ (2)

(1), (2) $\to 1 + \frac{3m-1}{4} = \frac{4+3m-1}{4} = \frac{3m+3}{4} = \frac{3}{4}(m+1)$ ✓

$(\frac{4}{10})^{2m-1} = (\frac{2}{5})^{2m-1} = (\frac{5}{2})^{-2m+1}$ ۲ ۸

$(\frac{125}{8})^{m^2} = ((\frac{5}{2})^3)^{m^2} = (\frac{5}{2})^{3m^2}$

$\left\{ (\frac{5}{2})^{-2m+1} = (\frac{5}{2})^{3m^2} \to -2m+1 = 3m^2 \right.$

$3m^2 + 2m - 1 = 0$

$\Delta = 4 - 4(3)(-1) = 16$

$\frac{-2\pm 4}{6} \begin{cases} \frac{1}{3} \Rightarrow (\frac{2}{5})^{-\frac{1}{3}} = (\frac{125}{8})^{\frac{1}{9}} \Rightarrow (\frac{5}{2})^{\frac{1}{3}} = (\frac{5}{2})^{\frac{1}{3}} ✓ \\ -1 \to \log_8^{-1} ✗ \end{cases}$

$\frac{1}{3} \to \log_8 4 \Rightarrow \log_{2^3} 2^2 = \frac{2}{3}\log_2 2 = \frac{2}{3}$ ✓

$\log_2 b = 2(1+a)$ (1) $\quad \log_2 3 = a \to \log_2 2 + \log_2 3 = 1 + a = \log_2 6$ ۲ ۹

$2\log_2 6 = 2(a+1) \to \log_2 36 = 2(a+1)$ (2) $\quad$ (1), (2) $\to 36 = b$ ✓

$\log(3(36)-8) = \log 100 = 2$ ✓

$\frac{-b}{-4a} = \frac{b}{4a} \to \frac{4a}{b} = \log_2 4 \to b = \frac{4a}{\log_2 4}$ $\left.\right\}$ $2a = \frac{4a}{\log_2 4} + c$ ۰/۵ ۱۰

$2a = b + c$

$c = 2a - \frac{4a}{\log_2 4} \Rightarrow \frac{c}{a} = 2 - \frac{4}{\log_2 4}$ $\quad (\frac{1}{\sqrt[8]{2}})^{\frac{c}{a}} = (2^{-\frac{1}{8}})^{2-\frac{4}{\log_2 4}} \to (2^{-\frac{1}{8}})^{2-\frac{2}{\log_2 2}}$

$2^{(-\frac{1}{4} + \frac{1}{2\log_2 2})}$

۱۰-

$$a = \frac{b+c}{2} \rightarrow b = 2a - c$$

$$\frac{1}{x_1 + x_2} = \frac{1}{S} = \frac{4a}{b} = \frac{\overset{2}{\cancel{4}}a}{2a-c} = \log 4 = \cancel{2}\log 2 \rightarrow \frac{2a}{2a-c} = \log 2$$

$$\xrightarrow{\text{معکوس}} \frac{2a-c}{2a} = \log_2 10 \rightarrow 1 - \log_2 10 = \frac{c}{2a} \rightarrow 2 - \log_2 100 = \frac{c}{a}$$

$$\left(2^{-\frac{1}{8}}\right)^{\frac{c}{a}} = \left(2^{\frac{c}{a}}\right)^{-\frac{1}{8}} = \left(2^{2-\log_2 100}\right)^{-\frac{1}{8}} = \left(\frac{4}{100}\right)^{-\frac{1}{8}} = 5^{\frac{1}{4}} = \boxed{\sqrt[4]{5}}$$