۱۴,۲۵

مهرشاد جابرزاده — یازدهم ب B — تکلیف ۲۳

نام و نام خانوادگی .... پاسخنامه تشریحی تکلیف شماره .... کلاس ....

۱ (۲)

$n^a = m$ و $m^b n^b = ...$

$m^b = (n^a)^b = n^{ab}$ ، $n^{ab}\times n^b = n^{2a}\times n \Rightarrow ab+b = 2a$ ، $\log_{mn}^{m^2n} = \log_{mn}^{mn} + \log_{mn}^{m} = b$

$\log_{mn}^{m^2n} = 1+\log_{n\times n^a}^{n^a} = 1+\frac{a}{a+1}$

با توجه به مثبت بودن (a) جواب ان گفت: $\frac{a}{a+1}$ همواره کوچکتر از یک هست

$b = 1+\frac{a}{a+1} \Rightarrow [b] = 1$

۲ (۱,۷۵)

الف) $x>0$ و $\frac{x}{\log_{\frac12}^x} > 0 \Rightarrow \log_{\frac12}^x > 0 \Rightarrow x<1$ و $x>0 \Rightarrow D_f = (0,1]$

$(0,1)$

مخرج کسر نمی‌تواند صفر باشد!

ب) $x^2-x-2>0$ ، $(x-2)(x+1)>0$ و $\sqrt{x^2-1} \to x^2-1\geq 0 \to x\geq 1 \cup x\leq -1$ و $x^2-1\neq 0 \Rightarrow x\neq \pm 1$

I $x>2$ II $x<-1$ ، $x>1$ ، $x<-1 \Rightarrow$

$D_f = \mathbb{R} - [-1,2]$

۳ (۲)

$2\log_x^a = \frac{2}{\log_a^x}$ و $\log_a^{\sqrt{x}} = \frac12\log_a^x \to \frac{2}{\log_a^9} + \frac12\log_a^9 = 2$ ، $\log_a^9 = t$

$\frac{2}{t}+\frac{t}{2} = 2 \Rightarrow (4+t^2 = 4t) \Rightarrow t^2+4-4t=0 = (t-2)^2=0 \to t=2$

$\log_a^9 = 2 \Rightarrow a^2 = 9 \Rightarrow a = 3$

۴ (۲)

$\log\frac{5}{2} = \log 5 - \log 2 = \log 10 - \log 2 - \log 2$ و $\log 9 = 2\log 3$ و $\log\frac{15}{2} = (\log 10 - \log 2) + \log 3$

$(1-\frac{3}{10}-\frac{3}{10})x^2 + (\frac{8}{10})x - (1-\frac{3}{10}+\frac{5}{10}) = (1-\frac{7}{10})$ ... $\Rightarrow (\frac{4}{10}x^2+\frac{8}{10}x-\frac{11}{10}=0)10$

$4x^2+8x-11=0 \Rightarrow x_1 = -\frac{22}{4}$ ، $x_2 = 1 \Rightarrow |x_2-x_1| = \frac{26}{4} = \frac{13}{2}$

۵ (۰,۵)

$\log_5^2 = \frac{1}{\log_2^5} = \frac{1}{\log_2^{10}-\log_2^2} = \frac12 \to \log_2^{10}-1 = 2 \Rightarrow \log_2^{10} = 3$

$\log_{10}^{14} = \frac{1}{\log_{14}^{10}}$

$\log_{14}^{10} = \frac{\log_2^{10}}{\log_2^{14}} = \frac{\log_2^5+\log_2^2}{\log_2^7+\log_2^2} = \frac{2+1}{2.8+1} = \frac{3}{3.8} = \frac{15}{19}$

$\log_2^5 = \frac{1}{\log_5^2} = 2$

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۶ — (۲)

$\log_6 3 = \frac{1}{\log_3 6} = \frac{1}{\log_3^{2\times3}} = \frac{1}{1+\log_3 2} = \frac{1}{1+\frac{1}{\log_2 3}} = \frac{1}{1+\frac{20}{16}} = \frac{13}{8} = \frac{8}{12}$

$\log_3 5 = \frac{\log_6 5}{\log_6 3} = \frac{\log_6 5}{\frac{8}{12}} = \frac{18}{15} \Rightarrow \log_6 5 = \frac{60}{60} = \frac{12}{12}$ $\Big\}$ $\log_{15} 6 = \frac{1}{\log_6 15}$, $\log_6 15 =$

$\log_6 (5\times3) = \log_6 5 + \log_6 3 = \frac{12}{12} + \frac{8}{12} = \frac{20}{12}$ $\log_{15} 6 = \frac{1}{\frac{20}{12}} = \frac{12}{20}$ ✓

۷ — (۱٫۵)

$\log_8 18 = m = \log_{2^3} 3^2\times2 \Rightarrow \frac{2}{3}\log_2 3 + \frac{1}{3}\log_2 2 = m$ $\log_{2^2} 12 = \log_{2^2} 2^2 + \log_2 3$

$t = \frac{3m-1}{2}$

$m = \frac{2}{3}\log_2 3 + \frac{1}{3}$, $\log_4 12 = 1 + \frac{\log_2 3}{2}$ $\to 2t+1 = 3m \to 1 + \frac{\log_2 3}{2} =$

$\log_2 3 = t$

$1 + \frac{1}{2}\left(\frac{3m-1}{2}\right) = \frac{3m+3}{4}$

$\log_4 4\times3 = \log_4 4 + \log_{2^2} 3$

۸ — (۲)

$\left(\frac{2}{5}\right)^{3x-1} = \left(\frac{5}{2}\right)^{4x^2}$, $\frac{5}{2} = \left(\frac{2}{5}\right)^{-1} \Longrightarrow \left(\frac{5}{2}\right)^{-1\times(3x-1)} = \left(\frac{5}{2}\right)^{4x^2} \Rightarrow -3x+1 = 4x^2$

$4x^2+3x-1=0 \to x_1=-1$, $x_2=\frac{1}{4}$, $9x+1 \to 9\times\frac{1}{4}+1 = 4+1 = (5)$

با جایگذاری این جواب می‌گیریم $x=-1$ غ ق ق log است

$\log_{2^2} 4 = \frac{2}{4}\times\log_2 2 = \frac{2}{4}$ ✓

۹ — (۲)

$1+a = \log_2 2 + \log_2 3 = \log_2 6 \to \log_8 b = \frac{2}{3}\times\log_2 6 = \log_8 36 \Rightarrow b=36$ ✓

$\log_8 36 = \log_8 b \Rightarrow b=36 \to 3b-8 = 3\times36-8 = 100$, $\log_{10} 100 = (2)$ ✓

(۲)

۱۰ — (۰٫۵)

مجموع ریشه‌های معادله درجه دوم $= \frac{-b}{-4a} = \log 4$ ✓ $\Rightarrow \frac{b}{4a} = 10000$, $2a = b+c$ ✓ $a = \frac{b+c}{2}$

$\frac{1}{\sqrt[8]{2}} = 2^{-\frac{1}{8}} \to \left(2^{-\frac{1}{8}}\right)^{\frac{c}{a}}$

$\log_{10} 4 = 2\log_{10} 2 = \frac{b}{4a}$

$4a = 2b+2c$

$\log_{10} 2 = \frac{b}{4(b+c)}$

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$$a = \frac{b+c}{2} \rightarrow b = 2a - c$$

$$\frac{1}{x_1 + x_2} = \frac{1}{S} = \frac{4a}{b} = \frac{2 \cancel{4a}}{2a - c} = \lg 4 = 2 \lg 2 \rightarrow \frac{2a}{2a-c} = \lg 2$$

معکوس $\rightarrow$ $\frac{2a-c}{2a} = \lg_2 10 \rightarrow 1 - \lg_2 10 = \frac{c}{2a} \rightarrow 2 - \lg_2 100 = \frac{c}{a}$

$$(2^{-\frac{1}{8}})^{\frac{c}{a}} = (2^{\frac{c}{a}})^{\frac{-1}{8}} = (2^{2 - \lg_2 100})^{\frac{-1}{8}} = (\frac{4}{100})^{\frac{-1}{8}} = 25^{\frac{1}{4}} = \boxed{\sqrt[4]{25}}$$