

$$\log_n^m = a \rightarrow m = n^a$$

$$\log_{mn}^{m^r n} = b \rightarrow \log_{n^a \cdot n}^{n^{ra} \cdot n} = b \rightarrow \log_{n^{a+1}}^{n^{ra+1}} = b \rightarrow \frac{ra+1}{a+1} = b$$

$$b = r - \frac{1}{a+1} \xrightarrow{0 < a < 1} [b] = \left[r - \frac{1}{a+1} \right] = [1]$$

۱

الف) $\log_{\frac{1}{r}} x \rightarrow x > 0$ ①

$$\log_{\frac{1}{r}} x = -\log_r x \xRightarrow{\text{رادیکال}} x < 1$$

$$\sqrt{\frac{x}{\log_{\frac{1}{r}} x}} \xrightarrow{\text{مخرج}} \frac{x}{\log_{\frac{1}{r}} x} > 0$$

ب) $\log_{\frac{1}{r}} (x^2 - x - 2) \rightarrow x^2 - x - 2 > 0$

$$x \rightarrow \begin{matrix} 2 \\ -1 \end{matrix} \quad \begin{matrix} + & - & + \\ - & + & - \end{matrix} \quad x < -1 \text{ یا } x > 2$$

$$\sqrt{x^2 - 1} + 1 \xrightarrow{\text{این بازه}} \text{همیشه مثبت}$$

$$\Rightarrow \sqrt{x^2 - 1} + 1 > 0 \Rightarrow x^2 - x - 2 > 0 \Rightarrow x < -1 \text{ یا } x > 2$$

$$x^2 - x - 2 = 0 \rightarrow \frac{1 \pm \sqrt{1+8}}{2} \Rightarrow x = \begin{cases} \frac{1+\sqrt{9}}{2} \\ \frac{1-\sqrt{9}}{2} \end{cases}$$

۲

$$x = 9 \rightarrow 2 \log_{\frac{a}{3}} 9 + \log_{\frac{a}{3}}^3 = 2 \Rightarrow \log_{\frac{a}{3}} 9 + \log_{\frac{a}{3}}^3 = 2$$

$$\frac{1}{\log_{\frac{a}{3}} 9} + \log_{\frac{a}{3}}^3 - 2 = 0 \Rightarrow \log_{\frac{a}{3}}^3 = t \Rightarrow \frac{1}{t} + t - 2 = 0$$

$$t^2 - 2t + 1 = 0 \Rightarrow (t-1)^2 = 0 \Rightarrow t = 1 \Rightarrow \log_{\frac{a}{3}}^3 = 1 \Rightarrow a = 3$$

۳

$$\text{تفاضل ریشه ها} = \frac{\sqrt{\Delta}}{|a|} \Rightarrow \Delta = ((\log 3^2)^2 + 4 \log 15 \cdot \log \frac{5}{3} = 4 \log 3 + 4 \log 5$$

$$\rightarrow 4(\log 3 + \log \frac{5}{3}) = 4(0.477 + \log 5 - \log 3) = 4.14$$

$$\Rightarrow \frac{\sqrt{4.14}}{\log 3} = \frac{\sqrt{4.14}}{0.477 - 0.176} = \frac{\sqrt{4.14}}{0.301} \approx \frac{2.03}{0.301} = \boxed{6.74}$$

۴

$$\log_{\frac{1}{14}} 10 = \log_{\frac{1}{\sqrt{2} \times 7}} 10 = \log_{\sqrt{2}} 10 + \log_7 10 \Rightarrow \frac{1}{\log_{\sqrt{2}} 10} + \frac{1}{\log_7 10} = \log_{\frac{1}{14}} 10$$

$$\rightarrow \frac{1}{2.18} + \frac{1}{0.15} = \frac{5}{14} + 2 = \boxed{\frac{33}{14}}$$

۵

$$\log_{1\Delta}^{\frac{r}{\Delta}} = \log_{\frac{r}{\Delta} \times \Delta}^{\frac{r}{\Delta} \times \Delta} = \log_{\frac{r}{\Delta}}^{\frac{r}{\Delta}} + \log_{\Delta}^{\frac{r}{\Delta}} \rightarrow \frac{1}{\log_{\frac{r}{\Delta}}^{\frac{r}{\Delta}}} + \frac{1}{\log_{\Delta}^{\frac{r}{\Delta}}} = \log_{1\Delta}^{\frac{r}{\Delta}}$$

$$\Rightarrow \frac{1}{\log_{\frac{r}{\Delta}}^{\frac{r}{\Delta}}} + \frac{1}{\log_{\Delta}^{\frac{r}{\Delta}}} = \frac{\Delta}{\Delta} + \frac{r}{r} = \frac{r+1}{r}$$

6

$$\log_{\frac{r}{\Delta}}^{\frac{r}{\Delta} \times \Delta} = m \Rightarrow \log_{\frac{r}{\Delta}}^{\frac{r}{\Delta}} + \log_{\Delta}^{\frac{r}{\Delta}} = m \Rightarrow \frac{1 + r \log_{\Delta}^{\frac{r}{\Delta}}}{r} = m$$

$$\Rightarrow \log_{\Delta}^{\frac{r}{\Delta}} = \frac{r(m-1)}{r}$$

7

$$\log_{\frac{r}{\Delta}}^{\frac{r}{\Delta}} = \log_{\frac{r}{\Delta} \times \Delta}^{\frac{r}{\Delta} \times \Delta} = 1 + \frac{1}{r} \log_{\Delta}^{\frac{r}{\Delta}} = 1 + \frac{1}{r} \cdot \frac{r(m-1)}{r} = \frac{r(m+1)}{r} = \frac{r}{r} (m+1)$$

$$\left(\frac{\Delta}{r}\right)^{1-rx} = \left(\frac{\Delta}{r}\right)^{rx} \rightarrow rx + rx - 1 = 0 \rightarrow x \rightarrow -1 \text{ or } \frac{1}{r}$$

$$\rightarrow \log_{\Delta}^{\left(4\left(\frac{1}{r}\right) + 1\right)} = \log_{\frac{r}{\Delta}}^{\frac{r}{\Delta}} = \frac{r}{r}$$

8

$$\log_{\Delta}^b = \frac{r}{r}(1+a) = \frac{r}{r}(1 + \log_{\Delta}^{\frac{r}{\Delta}}) \rightarrow \frac{r}{r} \log_{\Delta}^{\frac{r}{\Delta}} = \log_{\Delta}^b$$

$$\Rightarrow \log_{\frac{r}{\Delta}}^{\frac{r}{\Delta}} = \log_{\Delta}^b \Rightarrow b = \frac{r}{r} \text{ ①}$$

$$\log(rb-1) \xrightarrow{\text{①}} \log(101-1) = \log 100 = \frac{r}{r}$$

9

$$\frac{1}{\alpha + \beta} = \log_{\Delta}^{\frac{r}{\Delta}} \Rightarrow \frac{-b}{-ra} = \alpha + \beta = \log_{\Delta}^{\frac{r}{\Delta}} \Rightarrow \frac{b}{ra} = \frac{1}{r} \log_{\Delta}^{\frac{r}{\Delta}} = \log_{\Delta}^{\frac{r}{r}}$$

$$a = \frac{b+c}{r} \Rightarrow \frac{c}{a} = r - \frac{b}{a} \Rightarrow \left(\frac{1}{\sqrt{r}}\right)^{\frac{c}{a}} = r^{\frac{c}{ra}} = r^{\frac{1}{r} \left(r - \frac{b}{a}\right)}$$

$$= r^{\frac{1}{r}} \times r^{\frac{b}{ra}} = r^{\frac{1}{r}} \times r^{\frac{1}{r} \log_{\Delta}^{\frac{r}{\Delta}}} = r^{\frac{1}{r}} \times r^{\log_{\Delta}^{\frac{r}{r}}}$$

$$= \frac{1}{r^{\frac{1}{r}}} \times r^{\log_{\Delta}^{\frac{r}{r}}} = r^{\log_{\Delta}^{\frac{r}{r}}} = \sqrt[r]{\Delta}$$

10