

$$\log_n^m = a \rightarrow m = n^a$$

$$\log_{mn}^{m^r n} = b \rightarrow \log_{n^a \cdot n}^{n^{ra} \cdot n} = b \rightarrow \log_{n^{a+1}}^{n^{ra+1}} = b \rightarrow \frac{ra+1}{a+1} = b$$

$$b = r - \frac{1}{a+1} \xrightarrow{0 < a < 1} [b] = \left[ r - \frac{1}{a+1} \right] = [1] \quad \checkmark$$

(۲)  
۱

الف)  $\log_{\frac{1}{r}} x \rightarrow x > 0$  ①

$$\log_{\frac{1}{r}} x = -\log_r x \xRightarrow{\text{رادیكال}} x < 1 \quad \text{②}$$

$$\sqrt{\frac{x}{\log_{\frac{1}{r}} x}} \xrightarrow{\text{مخرج}} 0 < x < 1 \quad \checkmark$$

ب)  $\log_{\frac{x^2-x-2}{r}} \rightarrow x^2-x-2 > 0$  ③

$$x \rightarrow \begin{matrix} 2 \\ -1 \end{matrix} \quad x < -1 \text{ یا } x > 2$$

$$\sqrt{x^2-x-2} + 1 \xrightarrow{\text{این بازه}} \text{میشد مثبت}$$

$$x^2-x-2 > 0 \Rightarrow x^2-x-2 \geq 1$$

$$x^2-x-3=0 \rightarrow \frac{1 \pm \sqrt{13}}{2} \Rightarrow x \in \left( \frac{1-\sqrt{13}}{2}, \frac{1+\sqrt{13}}{2} \right)$$

(۱, ۵)  
۲

$$x=9 \rightarrow 2 \log_{\sqrt{r}} a + \log_a^{\sqrt{r}} = 2 \Rightarrow \log_{\sqrt{r}} a + \log_a^{\sqrt{r}} = 2$$

$$\frac{1}{\log_a^{\sqrt{r}}} + \log_a^{\sqrt{r}} - 2 = 0 \Rightarrow \log_a^{\sqrt{r}} = t \Rightarrow \frac{1}{t} + t - 2 = 0$$

$$t^2 - 2t + 1 = 0 \Rightarrow (t-1)^2 = 0 \Rightarrow t = 1 \Rightarrow \log_a^{\sqrt{r}} = 1 \Rightarrow a = \sqrt{r} \quad \checkmark$$

(۲)  
۳

$$\text{تفاضل ریشه ها} = \frac{\sqrt{\Delta}}{|a|} \Rightarrow \Delta = ((\log_{\sqrt{r}})^2 + 4 \log_{10} \cdot \log_{\frac{5}{r}} = 4 \log_{\sqrt{r}}^2 = 4 (\log_{\sqrt{r}})^2$$

$$\rightarrow 4 (\log_{\sqrt{r}} + \log_{\frac{10}{r}}) = 4 (0.14 + \log_{10} - \log_r) = 4.14$$

$$\Rightarrow \frac{\sqrt{4.14}}{\log_{\sqrt{r}}} = \frac{\sqrt{4.14}}{0.17 - 0.14} = \frac{\sqrt{4.14}}{0.03} \approx \frac{2.03}{0.03} = \boxed{67}$$

۴

$$\log_{\frac{10}{14}} = \log_{\sqrt{r} \times \frac{5}{2}} = \log_{\sqrt{r}} + \log_{\frac{5}{2}} \Rightarrow \frac{1}{\log_{\sqrt{r}}} + \frac{1}{\log_{\frac{5}{2}}} = \log_{\frac{10}{14}}$$

$$\rightarrow \frac{1}{2.14} + \frac{1}{0.15} = \frac{5}{14} + 2 = \boxed{\frac{33}{14}}$$

۵

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |    |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| $\log_{1\Delta}^4 = \log_{r \times \Delta}^{r \times r} = \cancel{\log_r^r} + \log_{\Delta}^r \rightarrow \frac{1}{\log_r^r} + \frac{1}{\log_{\Delta}^r} = \log_{1\Delta}^4$ $\Rightarrow \frac{1}{1/4} + \frac{1}{1/\Delta} = \frac{\Delta}{1} + \frac{r}{r} = \boxed{\frac{r+1}{r}}$                                                                                                                                                                                                                                                                                                                                                                 | 6  |
| $\log_{r^r}^{r \times r} = m \Rightarrow \log_{r^r}^r + \log_{r^r}^{r^r} = m \Rightarrow \frac{1 + r \log_r^r}{r^r} = m$ $\Rightarrow \log_r^r = \frac{r^r m - 1}{r}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 7  |
| $\log_{r^r}^{1r} = \log_{r^r}^{r^r \times r} = 1 + \frac{1}{r} \log_r^r = 1 + \frac{1}{r} \cdot \frac{r^r m - 1}{r} = \frac{r^r m + r}{r} = \frac{r}{r} (m+1)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | 8  |
| $\left(\frac{\Delta}{r}\right)^{1-rx} = \left(\frac{\Delta}{r}\right)^{rx} \rightarrow rx + rx - 1 = 0 \rightarrow x \rightarrow -1 \text{ or } \frac{1}{r}$ $\rightarrow \log_{\Delta}^{4\left(\frac{1}{r}\right)+1} = \log_{r^r}^{r^r} = \boxed{\frac{r}{r}}$                                                                                                                                                                                                                                                                                                                                                                                        | 9  |
| $\log_{\Delta}^b = \frac{r}{r}(1+a) = \frac{r}{r}(1+\log_r^r) \rightarrow \frac{r}{r} \log_r^r = \log_{\Delta}^b$ $\Rightarrow \log_{r^r}^{4r} = \log_{\Delta}^b \Rightarrow b = \underline{r^4} \text{ ①}$ $\log(r^b - 1) \xrightarrow{\text{①}} \log(101 - 1) = \log 100 = \boxed{r}$                                                                                                                                                                                                                                                                                                                                                                | 10 |
| $\frac{1}{\alpha + \beta} = \log r \Rightarrow \frac{-b}{-ra} = \alpha + \beta = \log \frac{1}{r} \Rightarrow \frac{b}{ra} = \frac{1}{r} \log \frac{1}{r} = \log \sqrt{\frac{1}{r}}$ $a = \frac{b+c}{r} \Rightarrow \frac{c}{a} = r - \frac{b}{a} \Rightarrow \left(\frac{1}{\sqrt{r}}\right)^{\frac{c}{a}} = r^{\frac{c}{ra}} = r^{\frac{1}{r} \left(r - \frac{b}{a}\right)}$ $= r^{\frac{1}{r}} \times r^{\frac{b}{ra}} = r^{\frac{1}{r}} \times r^{\frac{1}{r} \log \sqrt{\frac{1}{r}}} = r^{\frac{1}{r}} \times r^{\log \sqrt{\frac{1}{r}}}$ $= \frac{1}{r^{\frac{1}{r}}} \times \sqrt[r]{10} = \sqrt[r]{\frac{10}{r}} = \boxed{\sqrt[r]{\Delta}}$ | 11 |

ب)  $x^2 - x - 2 > 0 \rightarrow (-\infty, -1) \cup (2, +\infty)$  (1)

$x^2 - 1 \geq 0 \rightarrow \begin{cases} x \geq 1 \\ x \leq -1 \end{cases}$  (2)

$\xrightarrow{n} (-\infty, -1) \cup (2, +\infty)$

$\lg \frac{a}{r} = \lg a - \lg r = 0,7 - 0,4 = 0,3$        $\lg a = 1 - \lg r = 0,7$

$\lg 4 = 2 \lg r = 2(0,4) = 0,8$

$\lg 10 = \lg r + \lg a = 0,4 + 0,7 = 1,1$

$0,3x^2 + 0,8x - 1,1 = 0 \xrightarrow{a+b+c=0} \begin{cases} x_1 = 1 \\ x_2 = -\frac{1,1}{0,3} \end{cases} \rightarrow x_1 - x_2 = \boxed{\frac{14}{3}}$

$\lg \frac{10}{14} = \frac{\lg 10}{\lg 14} = \frac{\lg r + \lg a}{\lg r + \lg r} = \frac{r+1}{r,8+1} = \frac{r}{r,8} = \boxed{\frac{10}{14}}$

$\lg a = \frac{1}{\lg r} = r$

$\lg \frac{4}{10} = \frac{\lg 4}{\lg 10} = \frac{\lg r + \lg r}{\lg r + \lg a} = \frac{\frac{5}{8} + 1}{1 + 1,5} = \frac{\frac{13}{8}}{\frac{5}{2}} = \boxed{\frac{13}{20}}$

$\lg r = \frac{1}{\lg r} = \frac{5}{8}$

خواص تعاریف رویه بار دیده بخوان