

$$\log_n^m = a \rightarrow m = n^a$$

$$\log_{mn}^{m^2n} = \log_{n^a \times n}^{(n^a)^2 \times n} = \log_{n^{a+1}}^{n^{2a+1}} = \frac{2a+1}{a+1} = b$$

$$\rightarrow [b] = \left[\frac{2a+1}{a+1} \right] = \left[1 + \frac{a}{a+1} \right] \Rightarrow [b] = 1$$

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الف) $y = \sqrt{\frac{x}{\log \frac{1}{x}}}$ $\log \frac{1}{x} \rightarrow x > 0$

$$\log \frac{1}{x} \neq 0 \Rightarrow x \neq 1$$

باید نیز توان $\log \frac{1}{x} > 0$ معنی داشته باشد $\rightarrow \log \frac{1}{x} > 0 \rightarrow x < 1$ چون

توان معنی ۲ را از بین برد $\rightarrow D_f = (0, 1)$ $\rightarrow$ باید کسری باشد

ب) $x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0$

$$\rightarrow x \rightarrow (-\infty, -1) \cup (2, +\infty)$$

$$x^2 - 1 > 0 \rightarrow x^2 > 1$$

$$\rightarrow x > 1 \text{ یا } x < -1$$

$$\text{① و ②} \rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$$

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$$2 \log_a a + \log_a^3 = 2 \rightarrow \log_a a + \log_a^3 = 2 \rightarrow \log_a a + \frac{1}{\log_a} = 2$$

$$\downarrow$$

$$2 \log_a a = \log_a a$$

$$\rightarrow t + \frac{1}{t} = 2$$

$$\rightarrow t^2 - 2t + 1 = 0 \rightarrow (t-1)^2 = 0 \rightarrow t = 1$$

$$\rightarrow \log_a a = 1 \Rightarrow \boxed{a = 3}$$

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$$(\log 4 - \log 3) x^2 + (2 \log 3) x - (\log 3 + \log 4)$$

$$\left(\frac{\log 10}{1} - \frac{\log 2}{0.3} - \frac{\log 3}{0.4} \right) x^2 + (2 \log 3) x - \left(\frac{\log 10}{1} - \frac{\log 2}{0.3} + \frac{\log 3}{0.4} \right) = 0$$

$$0.3 x^2 + 0.1 x - 1.1 = 0 \xrightarrow{\times 10} 3x^2 + 1x - 11 = 0 \xrightarrow{\text{حالت خاص}} x = \frac{1}{3} \rightarrow 1 - \left(-\frac{11}{3} \right) = \boxed{\frac{14}{3}}$$

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$$\log_d^2 = 0.4 \rightarrow 2 = d^{0.4}$$

$$\log_d^4 = 2.8 \rightarrow 4 = d^{2.8}$$

$$\rightarrow 4 = (d^{0.4})^{2.8} = d^{1.12}$$

$$\log_{2.8}^4 d = \log_{d^{0.4}}^4 d = \log_{d^{1.12}}^4 d = \frac{1}{1.12} = \frac{1}{1.12} \approx 0.89$$

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$$\log_y^d = 1/d \rightarrow d = y^{1/d} \rightarrow d = (y^{1/y})^{1/d} = y^{y/f}$$

$$\log_y^y = 1/y \rightarrow y = y^{1/y}$$

$$\log_{1/d}^y = \log_{y \times y}^{y \times y} = \log_{y^{1/y} \times y^{1/y}}^{y \times y^{1/y}} = \log_{y^{1/y}}^{y^{1/y}} = \frac{y/y}{f} = \boxed{\frac{yd}{100}}$$

$$\log_f^y = \log_f^f + \log_f^y = 1 + \log_f^y \rightarrow 1 + \frac{y_{m-1}}{f} = \frac{y_{m-1} + f}{f} = \boxed{\frac{y_m + y}{f}}$$

$$\log_{1/y}^y = \log_{1/y}^y + \log_{1/y}^y = \frac{y}{y} \log_{1/y}^y + \frac{1}{y} = m$$

$$\log_y^y = \frac{y_{m-1}}{y}$$

$$\log_y^y = \frac{\log_f^y}{\log_f^y} = y \log_f^y \rightarrow \log_f^y = \frac{y_{m-1}}{f}$$

$$y^{yx-1} \times d^{-yx+1} = d^{yx} \times y^{-yx}$$

$$\rightarrow y^{yx-1} = y^{-yx} \rightarrow -yx^y = yx-1$$

$$\rightarrow yx^y + yx-1 = 0 \xrightarrow{\text{Cub}} x^{y-1} \times \frac{1}{y} \checkmark$$

$$x = \frac{1}{y} \rightarrow \log_{1/y}^{(y \times \frac{1}{y} + 1)} = \log_{1/y}^f = \boxed{\frac{y}{y}} \rightarrow \text{جواب}$$

$$\log_y^y = a$$

$$\log_{1/y}^b = \frac{1}{y} \log_y^b = \frac{y}{y} (1+a)$$

$$\rightarrow \log_{1/y}^b = y + ya = y + y \log_y^y = \log_y^f + \log_y^y = \log_{1/y}^{yy} \Rightarrow \boxed{b = yy}$$

$$\log(y^y b - 1) = \log 101 - 1 = \log 100 = \boxed{2} \rightarrow \text{جواب}$$

$$\alpha + \beta = \frac{-b}{-fa} = \frac{b}{fa} \rightarrow \frac{1}{\alpha + \beta} = \frac{fa}{b} = \log_f^f \rightarrow b = \frac{fa}{\log_f^f} \quad \frac{1}{\log_f^f} = \log_f^1$$

$$a = \frac{b+c}{y} \rightarrow ya = b+c \xrightarrow{b = \frac{fa}{\log_f^f}} ya = \frac{fa}{\log_f^f} + c \rightarrow a(y - \frac{f}{\log_f^f}) = c \rightarrow \frac{c}{a} = y - \frac{f}{\log_f^f}$$

$$\rightarrow \frac{c}{a} = y - f \log_f^1 = \log_f^f - y \log_f^1 = \log_f^{\frac{f}{y}} = \log_{1/y}^{\frac{1}{y}} = \log_{1/y}^{d^{-y}}$$

$$\left(\frac{1}{y}\right)^{\frac{c}{a}} = \left(y - \frac{1}{y}\right)^{\log_f^{\frac{f}{y}}} = \left(\frac{1}{y}\right)^{\log_{1/y}^{y-\frac{1}{y}}} = (d^{-y})^{-\frac{1}{y}} = d^{\frac{1}{f}} = \boxed{\sqrt[f]{d}} \rightarrow \text{جواب}$$