نام و نام خانوادگی ..آرشام پاینی.......... پاسخنامه تشریحی تکلیف شماره ..۲۳.... کلاس ...یازدهم........

**۱**

$\log_n m = a \to m = n^a$ $\quad \log_{mn} m^2 n = b \xrightarrow{m=n^a} \log_{n^{a+1}} n^{2a+1} = \frac{2a+1}{a+1}\log_n n$

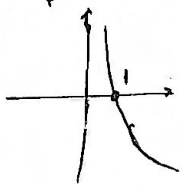
$= \frac{2a+1}{a+1} = b \xrightarrow{a>0} \frac{a+1}{a+1} < \frac{2a+1}{a+1} < \frac{2a+2}{a+1} \Rightarrow 1 < \frac{2a+1}{a+1} < 2$

$\Rightarrow 1 < b < 2 \Rightarrow [b] = 1$

**۲**

الف) $\log_{\frac{1}{2}} a \to a > 0$

$\log_{\frac{1}{2}} a = 0 \to a = 1$ ریشه ، $a \neq 1$



$\frac{a}{\log_{\frac{1}{2}} a} > 0 \to$ ت ن | + | − (0 ، 1)

$\Rightarrow D = (0, 1)$

ب) $a^2 - a - 2 > 0 \qquad\qquad \to a > 2 \cup a < -1$
$(a-2)(a+1) > 0 \qquad$ (I)

$\sqrt{a^2 - 1} \to a^2 - 1 \geq 0 \to a^2 \geq 1$

$\to a \geq 1 \cup a \leq -1$ (II)

$\xrightarrow{\text{اشتراک}} (-\infty, -1) \cup (2, +\infty)$

**۳**

$2\log_9 a = \log_9 a^2 = \log_{3^2} a^2 = \log_3 a \to \log_3 a + \log_a 3 = 2 \to \log_3 a = t$

$t + \frac{1}{t} = 2 \to t^2 + t - 2 = 0$ < $t = 1 \longrightarrow \log_3 a = 1 \Rightarrow a = 3$ ✓

$t = -2 \to \log_3 a = -2 \Rightarrow a = \frac{1}{9}$ ✗ در معادله صدق نمی‌کند

$\Rightarrow a = 3$

**۴**

$\log\frac{5}{3} = \log 5 - \log 3 = \log 10 - \log 2 - \log 3 = 1 - 0/3 - 0/4 = 0/3$

$\log 9 = 2\log 3 = 2 \times 0/4 = 0/8 \quad \log 15 = \log 5 + \log 3 = \log 10 - \log 2 + \log 3 = 1 - 0/3 + 0/4 = 1/1$

$\Rightarrow 0/3 x^2 + 0/8 x - 1/1 = 0 \xrightarrow{a+b+c=0} x = 1, \ x = \frac{-1/1}{0/3} \Rightarrow x_2 - x_1 = \frac{3}{3} + \frac{11}{3} = \frac{14}{3}$

**۵**

$\log_{14} 10 = \frac{\log_2 10}{\log_2 14} \longrightarrow \log_2 10 = \log_2 5 + \log_2 2 = \frac{1}{\log_5 2} + \log_2 2 = 2 + 1 = 3$

$\log_2 14 = \log_2 7 + \log_2 2 = 2/8 + 1 = 3/8$

$\Rightarrow \log_{14} 10 = \frac{\log_2 10}{\log_2 14} = \frac{3}{3/8}$

$$\log_{15}6=\frac{\log_3 6}{\log_3 15}\longrightarrow \log_3 6=\log_3 3+\log_3 2=1+\frac{1}{\log_2 3}=1+\frac{1}{1.6}=1.625$$

$$\longrightarrow \log_3 15=\log_3 3+\log_3 5=1+1.5=2.5$$

$$\Rightarrow \log_{15}6=\frac{\log_3 6}{\log_3 15}=\frac{1.625}{2.5}=0.65$$ ۶

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$$\log_8 18=\log_8 2+\log_8 9=\frac{1}{3}+\log_{2^3}3^2=\frac{1}{3}+\left(\frac{2}{3}\log_2 3\right)=m\longrightarrow \frac{2}{3}\log_2 3=\frac{2m}{3}$$

$$\Rightarrow \log_2 3=m\longrightarrow \log_4 12=\log_4 4+\log_4 3=\log_4 4+\frac{1}{2}\log_2 3$$

$$=1+\frac{1}{2}m=\frac{2+m}{2}$$ ۷

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$$\left(\frac{2}{5}\right)^{2x-1}=\left(\frac{5}{2}\right)^{3x^2}\Rightarrow 2x-1=-3x^2\longrightarrow 3x^2+2x-1=0 < \begin{matrix} x=-1 \ \times \\ x=\frac{1}{3} \ \checkmark \end{matrix}$$

$$9x+1>0\longrightarrow 9x>-1\longrightarrow x>\frac{-1}{9}$$

$$\Rightarrow \log_8(9x+1)=\log_8 4=\frac{2}{3}\log_2 2=\frac{2}{3}$$ ۸

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$$\log_8 b=\frac{2}{3}+\frac{2}{3}a\longrightarrow \frac{2}{3}a=\frac{2}{3}\log_2 3=\log_8 9$$

$$\longrightarrow \log_8 b-\log_8 9=\frac{2}{3}\longrightarrow \log_8\frac{b}{9}=\frac{2}{3}\longrightarrow \frac{b}{9}=8^{2/3}=4\longrightarrow b=36$$

$$\longrightarrow \log(3b-8)=\log 100=\boxed{2}$$ ۹

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$$S=\frac{-b}{-4a}=\frac{b}{4a}\longrightarrow \frac{1}{S}=\frac{4a}{b}=\log 4\longrightarrow 4a=b\log 4\longrightarrow 2a=b\times\frac{1}{2}\log 4=b\log 2$$

$a=\frac{b}{2}\log 2$ $\quad$ a واسطه حسابی $\longrightarrow \frac{b+c}{2}=a\longrightarrow b+c=2a\longrightarrow c=2a-b$

$$\longrightarrow c=b\log 2-b=b(\log 2-1)=b(\log 2-\log 10)=b(\log 0.2)\Rightarrow \frac{c}{a}=\frac{b(\log 0.2)}{\frac{b}{2}(\log 2)}$$ ۱۰

$$=\frac{\log_{10}0.2}{\log_{10}\sqrt{2}}=\log_{\sqrt{2}}0.2\Rightarrow\left(2^{-\frac{1}{8}}\right)^{\log_{\sqrt{2}}0.2}=(0.2)^{\log_{\sqrt{2}}2^{-\frac{1}{8}}}=(0.2)^{-\frac{1}{4}}=5^{\frac{1}{4}}=\sqrt[4]{5}$$